# MATH 162 PRACTICE TEST III

**1.** For each series below, determine absolute convergence, conditional convergence or divergence. Justify each answer.

$$(a)\quad \sum_{n=3}^{\infty} (-1)^n \frac{13}{(\ln n)^{13}}$$

$$(b)\quad \sum_{k=1}^{\infty} (-1)^k \frac{(k+3)(k^2+5)}{(k+13\ln k)^4}$$

$$(c)\quad \sum_{n=1}^{\infty} (-1)^n \frac{2^n}{\left(1+\frac{1}{n}\right)^{n^2}}$$

$$(d)\quad \sum_{n=1}^{\infty} (-1)^n \frac{1}{\ln(e^n+e^{-n})}$$

$$(e)\quad \sum_{n=1}^{\infty} (-1)^n \frac{n^{13}}{(n+13)!}$$

**2.** For each power series below, determine the *radius of convergence* and the *interval of convergence*. Study the behavior of each power series at the *endpoints*.

$(a)\quad \sum_{n=1}^{\infty} \frac{13^n}{n(n+13)} x^n$

(b) $\sum_{n=1}^{\infty} \frac{1}{n(n+3)(n+11)} (x-4)^n$

(c) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{3n+7}} (x+13)^n$

**3.** (a) Find the 3rd order Maclaurin polynomial of *cosh x*.

(b) Find the 5th order Taylor polynomial of *cos x* centered at x = π/2.

**4.** Find the 4th order Taylor polynomial of $e^x$ centered at x = 2.

**5.** Find the 3rd order Maclaurin polynomial of

$f(x) = 4 + (x+13)^2 + (x+13)^3$

**6.** By differentiating the power series expansion of y = 1/(1 – x), find the value of

$$\sum_{k=0}^{\infty} \frac{k}{13^k}$$

**7.** Find the *first five* non-zero terms of the Maclaurin series expansion of

$$h(x) = (1 + 2x^2)\, e^{3x}.$$

**8**. Let $f(x) = x^8 e^{5x}$. Compute $f^{(100)}(0)$. Do not simplify your answer.

***9.*** Find the radius of convergence of the power series:

$$\sum_{n=1}^{\infty} \frac{n!}{(1)(3)(5)(7)\ldots(2n-1)} x^n$$

***10.*** Find the radius of convergence of the power series:

$$\sum_{n=0}^{\infty} n!\, x^{2^n}$$

***11.*** Find the radius of convergence of the power series:

$$\sum_{n=1}^{\infty} \frac{1}{(\ln n)^n} x^n$$

**12 .** *Without using* l'Hôpital's rule, calculate the following limit. Show your work!

$$\lim_{t \to 0} \frac{te^{4t} - \sin(3t) + 2t - 4t^2}{t^3}$$

13. Let $G(x) = x^3 \cosh(3x)$. Using an appropriate Maclaurin series, compute $G^{(2013)}(0)$. (Do not try to simplify your answer.)

14. Compute:

$$\int \frac{x^2 + x + 4}{x^2 + 1} dx$$

**15**. (a) Express $\left(\frac{13+i}{1+i}\right)(1-2i)+4+5i$ as a complex number of the form a + bi.

(b) By expressing -1 as an appropriate complex power of *e*, calculate the five fifth roots of -*i*. Express your answers in the form a + bi.

(c) Using Euler's formula, express cos(4x) in terms of cos x and sin x.

**16**. Give the correct *form* (for example: A/(x+2) + B/(x –13)) for the partial fraction decomposition of the following rational functions. *Do not solve for A, B, C, …*

$$\frac{x^7 + 5x^3 + 9x + 13}{(x^2 - 2x - 15)^2 (x^2 + 2x + 15)^2}$$

**17.** Integrate each of the following functions:

$(a)$ $\quad \dfrac{1}{x^2 - 2x + 1}$

$(b)$ $\quad \tan^7 x$

$(c)\quad \dfrac{x}{x^4+4}$

$(d)\quad \dfrac{4x^3-20x}{x^4-10x^2+9}$

$(e)\quad \tan^6 2x$

$(f)\quad \sec^4 x$

$(g)\quad (\sec x)^{11/2}\tan x$

$(h)\quad (\sin 10x)(\sin 5x)$

$(i)\quad \sin^9 x\cos^{11} x$

$(j)\quad (\cos x)(\sin(\sin x))$

18. Express each of the following as a trigonometric integral. Do not evaluate.

$(a)\quad \int \dfrac{x^7}{x^2+1}\,dx$

$$(b) \quad \int x^9 \sqrt{x^2+1}\, dx$$

$$(c) \quad \int \frac{x^5}{\sqrt{1-x^2}}\, dx$$

$$(d) \quad \int \frac{\sqrt{x^2-1}}{x^8}\, dx$$

19. Using division of power series, find the first three non-zero terms of the Maclaurin series expansion of

$$f(x) = \frac{e^{2x}+1}{\cos x}$$

20. Using multiplication of power series, find the first four non-zero terms of the Maclaurin series expansion of

$$g(x) = e^{x^2}(1+x^2+x^3)$$

21. Determine the *interval of convergence* of the following power series. (You need not study end-point behavior.)

$$\sum_{n=1}^{\infty} \frac{n^{13}\, 13^n}{\sqrt{n+2012}} (x-13)^n$$

22. Using substitution (or any other method that you prefer), evaluate each of the following integrals:

$$(a) \quad \int \frac{\tan(\ln x)}{x}\, dx$$

$$(b) \quad \int \sqrt{x}\, \sin(2x^{3/2})\; dx$$

$$(c) \quad \int e^x \sec^2(e^x - 13)\, dx$$

$$(d) \quad \int \sec x \tan x \sqrt{1 + \sec x}\; dx$$

$$(e) \quad \int \sqrt{a + b\sqrt{c + x}}\, dx$$

$$(f) \quad \int \frac{e^x \arcsin\sqrt{x}}{2\sqrt{x - x^2}}\, dx$$

$$(g) \quad \int \frac{dx}{\sqrt{\arctan x}\;\, (1 + x^2)}$$

**23.** Solve each of the following initial value problems:

(a) $dy/dt = (\ln t)^3/t$, $y(3) = 13$

(b) $dy/dx = \sec^2 x + \tan^2 3x + 14$, $y(0) = 88$

(c) $ds/dt = t \cos t + 1 + \sin t$, $y(\pi/2) = 2013$

24. Analyze the behavior of the series

$$\sum_{n=1}^{\infty} \frac{\sqrt{n^2+4}}{\left(n^{1/3}+2013\right)^5}$$

*As far as the laws of mathematics refer to reality, they are not certain; as far as they are certain, they do not refer to reality.*

- Albert Einstein