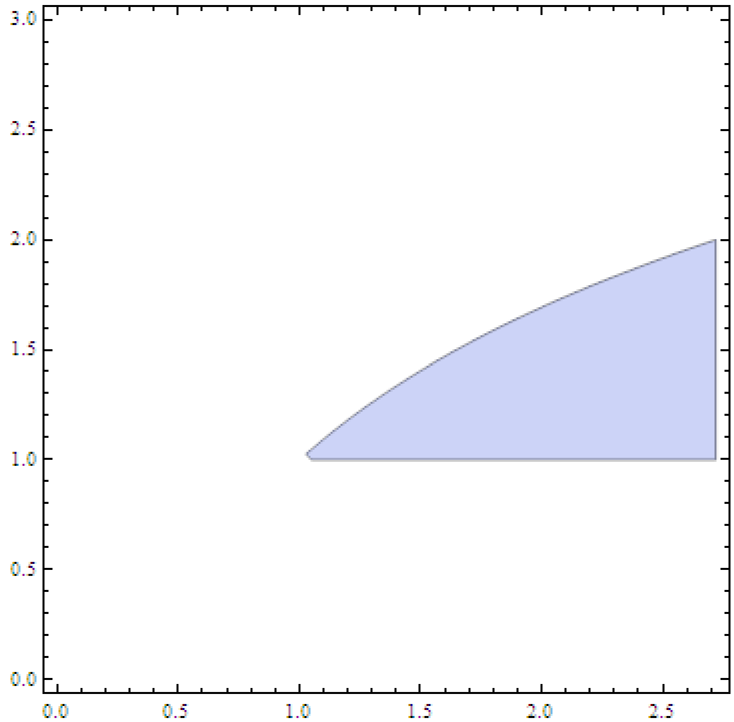
MATH 162 SOLUTIONS: QUIZ I

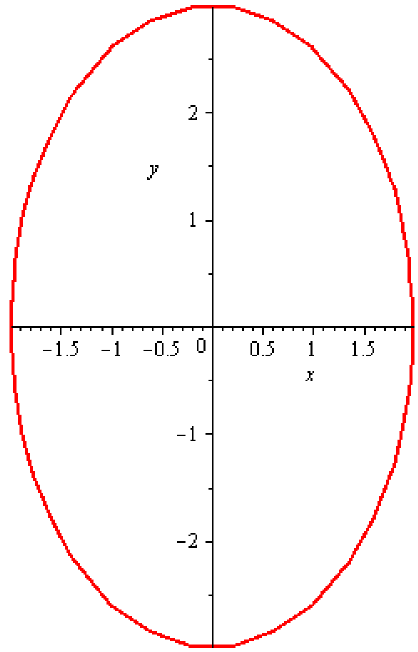
1. Sketch the region in the first quadrant bounded by the curve $y = 1 + \ln x$ and the lines $x = e$, and $y = 1$. This region is rotated about the line $x = -7$. Using the shell method, write a definite integral that expresses the volume of this solid of revolution. *Do not evaluate.*



To use shells, we fix x. Consider a thin vertical rectangle between $x = 1$ and $x = e$. The thickness of the rectangle is Δx. The area of the shell associated with this rectangle is $A(x) = 2\pi(x-(-7))\ln x$. Thus the total volume of the solid is:

$$V = \int_1^e A(x)\,dx = 2\pi\int_1^e (x+7)\ln x\,dx$$

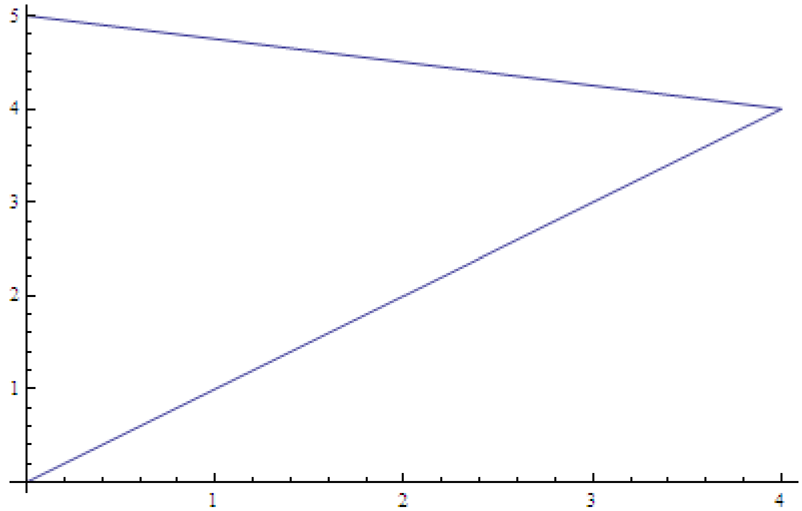
2. The base of a certain solid is an elliptical region given by the inequality $9x^2 + 4y^2 \leq 36$. Cross-sections perpendicular to the y-axis are semicircles. Express the volume of the solid as a definite integral. *Do not evaluate.*



Consider a thin horizontal rectangle between y=-3 and y=3. The thickness of the rectangle is Δy. The area of the semicircle associated with this rectangle is $(\pi/2)x^2$, where x = sqrt $((36 - 4y^2)/9)$. Thus the total volume of the solid is:

$$V = \int_{-3}^{3} \frac{1}{2}\pi\left(\sqrt{\frac{36-4y^2}{9}}\right)^2 dy = \frac{\pi}{18}\int_{-3}^{3} (36-4y^2)\,dy$$

3. Let **T** be the triangular region with vertices (0, 0), (4, 4) and (0, 5). Suppose that **T** is rotated about the axis y = -8. Using the washer method, write a definite integral that expresses the volume of this solid of revolution. *Do not evaluate.*

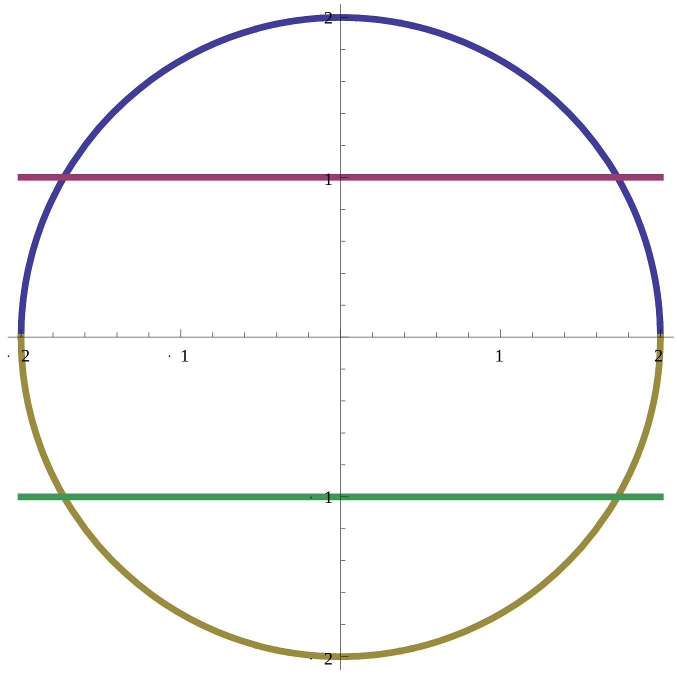


The equation of the line joining (0,5) and (4,4) is y = 5 – x/4 and the equation of the line joining (0,0) and (4,4) is y = x.
To use washers, we fix the value of x (between 0 and 4). Consider a corresponding vertical rectangle with width Δx. The outer radius of the washer is 5 – x/4 – (-8) = 13 – (x/4) and the inner radius of the washer is x + 8. Hence the volume of the solid is:

$$V = \int_0^4 \pi\left(\left(13 - \frac{x}{4}\right)^2 - (x+8)^2\right) dx$$

Extra Credit:

A round hole of length *2h* is bored through the center of a solid sphere of radius *R*. (Assume that h < R). Find the volume of the solid that remains.



If c denotes the radius of the hole, then $c^2 + h^2 = R^2$. *Consider the region in the upper half-plane bounded by the circle* $x^2 + y^2 = R^2$ *and the line* $y = c$. *Rotate this region about the x-axis. Consider a thin horizontal rectangle y-units above the axis where* $c \le y \le R$ *Rotate this rectangle about the x-axis to obtain a shell. The volume of this shell is* $2\pi y(2x)\Delta y$, *where* $x = (R^2 - y^2)^{1/2}$. *Thus the total volume of this solid of revolution is given by:*

$$V = \int_c^R 2\pi\, y(2x)\, dy = 4\pi \int_c^R y\left(\sqrt{R^2 - y^2}\right) dy =$$

$$\frac{4}{3}\pi\left(R^2 - c^2\right)^{\frac{3}{2}} = \frac{4}{3}\pi\left(h^2\right)^{\frac{3}{2}} = \frac{4}{3}\pi\ h^3$$

Notice the remarkable fact that this volume does not depend upon the radius of the sphere!

The Volume of Revolution Around the Horizontal Axis Between
f(x) = 3
and
g(x) = (25-x^2)^(1/2)
on the Interval [-4, 4]