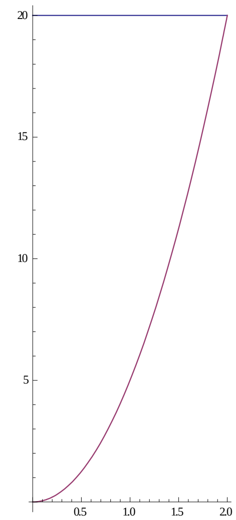
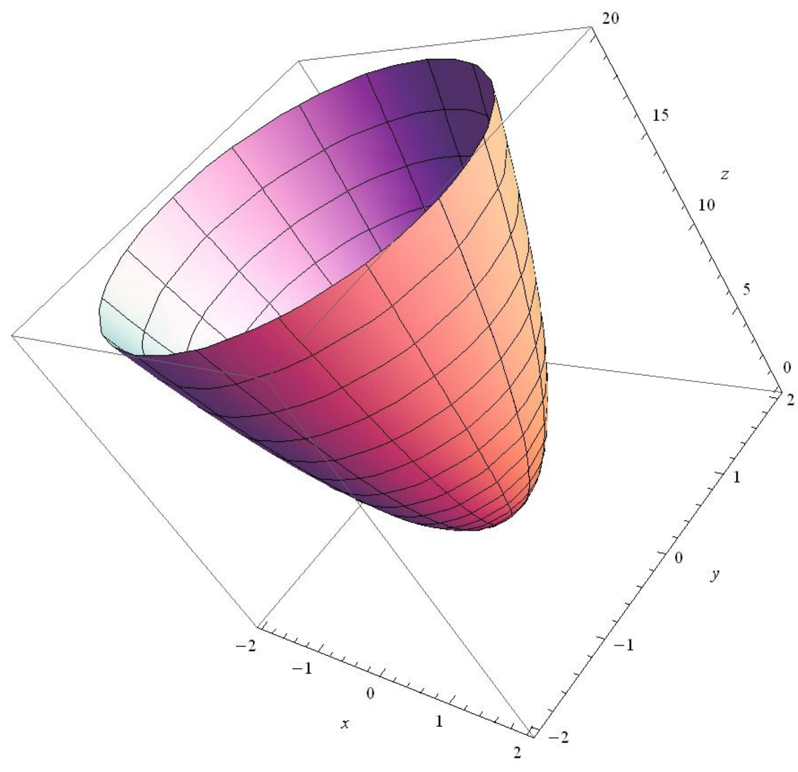
SOLUTIONS: QUIZ II

1. A tank is designed by revolving the parabola $y = 2x^2$, $0 \le x \le 2$, about the y-axis. The tank, with dimensions in feet, is filled with fluid weighing 41 lbs/ft^3. How much *work* will it take to empty the tank by pumping water to the tank's top? Give your answer to the nearest ft-lb. *Sketch!*





Let y denote any point in the interval [0, 20]. Consider a horizontal slice (passing through y) of our solid that has thickness Δy. The area of the slice (circle with radius $x = (y/5)^{1/2}$) is $\pi x^2 = \pi y/5$.

Hence the volume of the slice is $(\pi y/5)\Delta y$ (cubic feet). The weight of this slice is $(\pi y/5)(\Delta y)(62.4)$ lbs.

This slice is $(20 - y)$ feet from the top of the tank. Thus the work done in moving this slice to the top of the tank is:

$$\Delta W = (20 - y)(\pi y/5)(62.4)\Delta y$$

Summing and taking the limit as $\Delta y \to 0$, we obtain:

$$W = \int_0^{20} (20-y)\frac{\pi y}{5}(62.4)\,dy = \frac{62.4\pi}{5}\int_0^{20}(20-y)y\,dy = 39.2\left(10y^2 - \frac{y^3}{3}\right)\Bigg|_{y=0}^{20}$$

$$= 39.2\left(4000 - \frac{8000}{3}\right) = 39.2(4000)\frac{1}{3} = 52{,}276 \;\; ft - lbs.$$

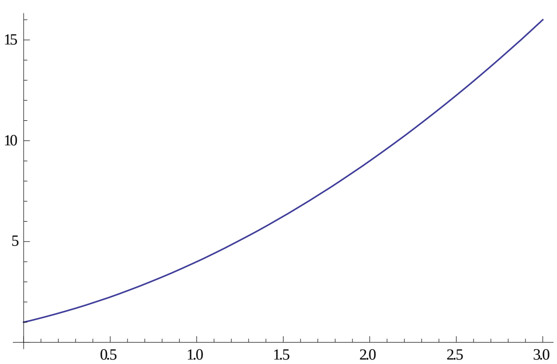
2. Consider the parameterized curve: $x = e^t - 1$, $y = e^{2t}$, where $t \geq 0$, describing the position in the xy-plane of Mehitabel, the cat, at time *t*.

(a) By eliminating the parameter, *t*, express *y* as a function of *x*.

Solution: Since $x = e^t - 1$, $e^t = x + 1$. Hence $y = e^{2t} = (x + 1)^2$.

(b) *Sketch* the parameterized curve, using an arrow to indicate the direction of Mehitabel's journey. Also, indicate the birthplace of Mehitabel.

Solution: When $t = 0$, $x = 0$ and $y = 1$. Thus Mehitabel is born at the point (0, 1) in the xy-plane. As $t > 0$ increases, both x and y increase. Thus Mehitabel's direction of motion is northwest.



3. Compute the *arc length* of the parameterized curve

$$x(t) = e^t + e^{-t}$$
$$y(t) = 5 - 2t$$
$$where\ 0 \le t \le 1$$

Here you should evaluate the integral and give a numerical answer. (You may use your calculator to check your work, but not as a substitute for evaluating the integral.)

Since dx/dt = e^t – e^{-t} and dy/dt = -2,

$$s = \int_0^1 \sqrt{(dx/dt)^2 + (dy/dt)^2}\ dt = \int_0^1 \sqrt{(e^t - e^{-t})^2 + (-2)^2}\ dt =$$

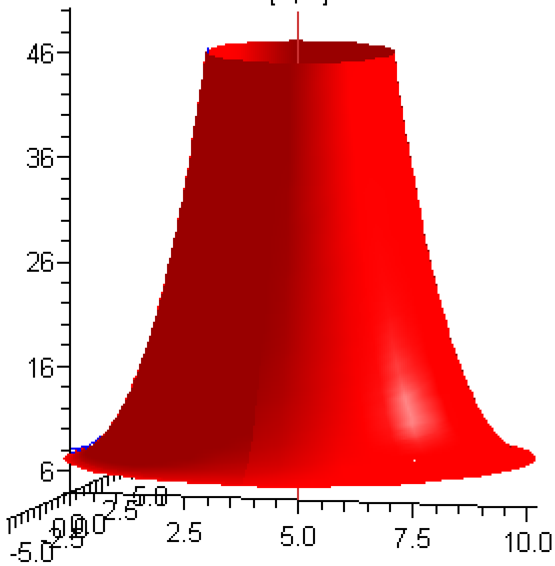
$$\int_0^1 \sqrt{e^{2t} - 2 + e^{-2t} + 4}\ dt = \int_0^1 \sqrt{e^{2t} + 2 + e^{-2t}}\ dt =$$

$$\int_0^1 \sqrt{(e^t + e^{-t})^2}\ dt = \int_0^1 (e^t + e^{-t})\ dt =$$

$$\left(e^t - e^{-t}\right)\Bigg|_0^1 = e - e^{-1} - (1-1) = e - e^{-1} \approx 2.35$$

4. The curve given by $y = \left(x^2 + 4\right)^{\frac{3}{2}}$ $for\ 0 \le x \le 3$ is rotated about the axis x = 5. Express the *area* of the generated surface as a Riemann integral. *Sketch! Do not evaluate the integral.*

The Volume of Revolution Around the Line x = 5 of
f(x) = (x^2+4)^(3/2)
on the Interval [0, 3]



Note that ρ, the radius of revolution, is 5 – x. Thus

$$S = \int_{x=0}^{x=3} 2\pi\rho\, ds = 2\pi \int_{x=0}^{x=3} (5-x)\sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx = 2\pi \int_0^3 (5-x)\sqrt{1+\left(\frac{3}{2}(x^2+4)^{\frac{1}{2}})2x\right)^2}\, dx =$$

$$2\pi \int_0^3 (5-x)\sqrt{1+9x^2(x^2+4)}\, dx = 2\pi \int_0^3 (5-x)\sqrt{9x^4+36x^2+1}\, dx$$

5. Albertine, a rock climber, is about to haul up 80 newtons (about 18 lbs) of equipment that has been hanging beneath her on 50 meters of rope that weighs 0.8 newton per meter. How much *work* will it take? Express your answer to the nearest joule.

Assume that Albertine's (fixed) position on the y-axis is y = 50 and that the equipment initially is at the origin. Then the work required to haul up the equipment alone is (50)(80) = 4000 joules. To haul up the rope, the work required is:

$$W = \int_0^{50} 0.8(50 - y)\, dy = 0.8\left(50y - \frac{y^2}{2}\right)\Bigg|_0^{50} = 1000 \;\; joules$$

Thus the total amount of work performed in this task is 5000 joules.