# Solutions: QUIZ IV

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|  | ***1.*** Select *any three* of the following four integrals. For each improper integral that you select, determine convergence or divergence. *Justify your answers!* (You may answer all four for extra credit.) *This integral diverges since:**Note that the dominant term in the numerator is 1 (not x15).* *This integral converges because:**Using the p-test, we know that  converges. Hence, by the comparison test,*  *converges*.*This integral diverges because:* *since cos(2c)→ 0+ as c → (/4)-.**This integral converges because, by definition:**and each of these two integrals converges (by virtue of the Comparison Test):**For 0 < x ≤ 1:**and by the p-test for integrals of type II,**For x ≥ 1:**and by the p-test for integrals of type I,****2.*** For each of the following *sequences*, determine *convergence* or *divergence*. In the case of convergence, find the *limit* of the sequence. Briefly justify each answer. *(Select any 7 of the 8 sequences. For extra credit, you may solve all eight.)**Since n = o(n!) and ln n = o(n), {an} converges and its limit is 0.* *Using the fact that the limit of the sum of two convergent sequences is the sum of their limits, we have:**Thus the sequence {bn } converges and its limit is e.**Since -1 ≤ sin(4n) ≤ 1, we have:**Applying the Squeeze Theorem, we conclude that {cn} converges to 0.**Observing that dn ≥ nn/ (n14 + n14) = ½ nn/ n14 → ∞ as n → ∞, we conclude that dn is unbounded, and thus divergent.* (e) en = (-1)n cos(1/n)*First note that, as n → ∞, cos(1/n) → cos 0 = 1.* *Thus for large n, en is approximately (-1)n which is a divergent sequence.**By selecting the dominant terms, we have:**Hence we conclude that {fn} converges to 12/5.*(g) gn = arctan (ln(n))*As n → ∞, ln n → ∞. and hence:**Thus the sequence {gn } converges and its limit is /2.**Rationalizing this expression:**Thus the sequence {hn } converges and its limit is 3.* |