**Solutions: QUIZ V**

1. Determine the *interval of convergence* of each of the following power series. Show your work! (You *need not* study end-point behavior.)

(a) 

*Applying the Ratio Test:*



*Thus the series converges absolutely when 7|x+13| < 1.*

*This is equivalent to |x+13| < 1/7. Hence the interval of convergence is*

*(-13 – 1/7, -13 + 1/7) and the radius of convergence is 1/7.*

 (b) 

*Applying the Ratio Test:*



*Hence the series converges for all real numbers x. The interval of convergence is thus (-∞, ∞).*

2. For each of the following numerical series, determine absolute convergence, conditional convergence or divergence. *Justify your answers.*

(a) 

*Note that, as n → ∞:*



*Thus, since an does not converge to 0, the n th term Test for Divergence implies that our series diverges.*

 (b) 

*Since n ln(n4) = 4n ln n:*



*Hence our original series does not converge absolutely. However, it does converge conditionally because the Leibniz– Cauchy Theorem is applicable:*

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(c) 

*Applying the Root Test we find that the series converges absolutely:*



(d) 

*Applying the Ratio Test, we see that the series converges absolutely:*

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***Extra Credit:*** For the following numerical series, determine absolute convergence, conditional convergence or divergence. *Justify your answer.*



*We claim that the series converges absolutely.*

*Since*

**

*we have*

**

*and thus, for large n:*

**

*or equivalently:*

**

*So:*



*Using the Comparison Test, we find that our series converges absolutely since Σ 1/n2 converges (by the p-test).*

*Alternatively, sin(1/n) ~ 1/n. So (sin(1/n))/n ~ 1/n2. Now use the Limit Comparison Theorem.*

*If you disregard the very simplest cases, there is in all of mathematics not a single infinite series whose sum has been rigorously determined. In other words, the most important parts of mathematics stand without a foundation.*

- Niels H. Abel(1802 - 1829)