# Solutions: QUIZ VI

**1.** Find the 5th order Taylor polynomial of **f(x) = ln x** centered at x = .

*Solution:*

*Since f(x) = ln x, we find that:*

*f′(x) = x-1*

*f′′(x) = – x-2*

*f(3)(x) = 2! x-3*

*f(4)(x) = – 3! x-4*

*f(5)(x) = 4! x-5*

*Hence f(3) = ln 3, f′(3) = 3-1, f′′(3) = – 3-2 , f(3)(3) = 2! 3-3, f(4)(3) = – 3! 3-4 and f(5)(3) = 4! 3-5.*

*So the 5th order Taylor polynomial of ln x centered at x = 3 is:*



2. Without differentiating, find a Maclaurin polynomial for the function **h(x) = arctan(4x)**

that has exactly *four* non-zero terms.

*Hint:* Begin with the expansion of 1/(1 – t) and then replace *t* by –t2.

*Solution:*



*Replacing t by t2:*



*Integration produces:*



*Thus, the first four non-zero terms of the Maclaurin expansion are:*



**3.**Find the 2nd order Maclaurin polynomial of **G(x) = sec 3x.**

*Solution:*

*Since G(x) = sec(3x), we have:*

*G′(x) = 3 sec(3x)tan(3x)*

*G′′(x) = 3{ sec(3x)sec2(3x) 3 + tan(3x) 3 sec(3x) tan(3x)} = 9{sec3(3x) + sec(3x) tan2(3x)}*

*Hence G(0) = 1, G′(0) = 0, and G′′(0) = 9.*

*So the 2nd order Maclaurin polynomial of sec(3x) is*

***P2(x) = 1 + (9/2!) x2 = 1 + (9/2) x2***

**4**. Find the 4th order Taylor polynomial centered at **x = 1** of the function **F(x) = (x – 4)4**.

*Solution:*

*Note that F(1) = 81, F′(1) = -108, F′′(1) = 108, F′′′(1) = -72 and F(4)(1)=24.*

*Thus the 4th degree Taylor polynomial is:*



**5**. Suppose that the Maclaurin series of **g(x) = arcsinh(x)** is given by:



Find g(5)(0).

*Solution:*

*We know that the coefficient of x5 in the Maclaurin series expansion is g(5)(0)/5!*

*Now the exponent is 5 when 2n + 1 = 5. That is, when n = 2.*

*So the coefficient of x5 in the given series expansion is (-1)2(4!)/{42) (2!)2(5)} which reduces to 3/40. Setting 3/40 = g(5)(0)/5!, we obtain: g(5)(0) = 3(5!)/40 =* ***9***

*Extra Credit*

Express the following integral as an infinite series.



*Solution:*

