# MATH 162 Solutions: Quiz VII

**1.** *Without using* l'Hôpital's rule, calculate the following limit. Show your work!



*Solution:*

*Using Maclaurin series:*



1. Let G(x) = x8 sin (3x). Using an appropriate Maclaurin series, compute G(2013)(0). *(Do not try to simplify your answer.)*

*Beginning with the Maclaurin series for sin t and then replacing t by 3x:*



*Now, multiplying by x8 yields:*



*Now, the general Maclaurin series of G(x) is:*

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*Thus the coefficient of x2013 is G(2013)(0) / 2013!*

*Now the series for x8 sin (3x) has coefficient of x2013 occur when 2n+9=2013, that is, when n =1002. Thus this coefficient is:*

 *(-1)1002 ∙32005 / 2005! = 32005 / 2005!*

*Equating G(2013)(0) / 2013! with 32005 / 2005!, we find that:*

*G(2013)(0) =* ***32005 (2013!)/ (2005!)***

**3**. (a) Express  as a complex number of the form a + bi.

*Solution:*



(b) By expressing -1 as an appropriate complex power of *e*, calculate the four fourth roots of -1.

*Since – 1 = ei = e3i = e5i = e7i, the four root of -1 are:*



(c) Express as a number in the form a + bi.



*Using Euler’s formula:*



**4.** Using Euler’s identity, express cos(5x) in terms of cos x and sin x.

*Solution:*

*Since exi = cos x + i sin x, we have:*

*e5xi = (cos x + i sin x)5 = (cos x)5 + 5(cos x)4 i sin x + 10 (cos x)3 i2 (sin x)2 + 10 (cos x)2 i3 (sin x)3 + 5 cos x (i4) (sin x)4 + i5 (sin x)5 = (cos x)5 + 5 (cos x)4 (sin x) i– 10 (cos x)3(sin x)2 – 10 (cos x)2(sin x)3 i + 5cos x (sin x)4 + (sin x)5 i*

*But Euler’s formula tells us that e5xi = cos(5x) + i sin(5x).*

*Thus cos(5x) = cos5 x – 10 cos3 x sin2 x + 5 cos x sin4 x*

**5.** Suppose that the 4th degree Maclaurin polynomial of f(x) is

1 + 2x – 3x2 + x3 – 3x4 and that the 4th degree Maclaurin polynomial of g(x) is

1 – x + x2 – x3 – 5x4.

1. Find the *first four non-zero* terms of the Maclaurin series of fg.

*Answer: 1 + x – 4x2 + 5x3*

1. Find the *first four non-zero* terms of the Maclaurin series of f/g.

*Answer: 1 + 3x – x2 – 2x3*

**Extra Credit:**

Find the *first 4 non-zero terms* in the Maclaurin series expansion of arcsin (x3).

(*Hint:* Begin by calculating the first few terms of the Maclaurin (or binomial) series for (1 – t)-1/2.)

*Solution:*

*Using the definition of the Maclaurin series, or the binomial expansion, we obtain:*



*Replacing t by t2 yields:*



*Integration produces:*



*Thus:*

