MATH 162 SOLUTIONS: QUIZ VIII

1. [15 pts] Give the correct *form* (for example: A/(x+2) + B/(x –13) for the partial fraction decomposition of each of the following rational functions. *Do not solve for A, B, C, …*

(a) $\dfrac{3x+5}{(x-1)(x-5)(x-13)}$

Since the roots of the denominator are unique:

$$\frac{A}{x-1}+\frac{B}{x-5}+\frac{C}{x-13}$$

(b) $\dfrac{x^5+5x^3+3x-11}{(x-1)^2(x+4)x^3}$

Note that this rational function is proper.

$$\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+4}+\frac{D}{x}+\frac{E}{x^2}+\frac{F}{x^3}$$

(c) $\dfrac{1}{(x^2-9)(x^2+5)^2}$

Note that the denominator can be factored further.

$$\frac{A}{x+3}+\frac{B}{x-3}+\frac{Cx+D}{x^2+5}+\frac{Ex+F}{(x^2+5)^2}$$

2. [50 pts] Answer any five of the following problems. You may answer all six to earn extra credit. Integrate the following functions:

$(a)\quad \tan^5 x$

Solution:

$$\int \tan^5 x\, dx = \int \left(\tan^3 x\right)\left(\sec^2 x - 1\right) dx = \int \left(\tan^3 x\right)\left(\sec^2 x\right) dx - \int \left(\tan^3 x\right) dx = \frac{\tan^4 x}{4} - \int \tan^3 x\, dx =$$

$$\frac{\tan^4 x}{4} - \int \tan^3 x\, dx = \frac{\tan^4 x}{4} - \int (\tan x)\left(\sec^2 x - 1\right) dx = \frac{\tan^4 x}{4} - \int (\tan x)\left(\sec^2 x\right) dx + \int \tan x\, dx =$$

$$\frac{\tan^4 x}{4} - \frac{\tan^3 x}{3} - \ln|\cos x| + C$$

(b) $\sec^{2013} x \tan x$ (revised)

Solution:

$$\int \sec^{2013} x \tan x\, dx = \int \left(\sec^{2012} x\right)\left(\sec x \tan x\right) dx = \frac{\sec^{2013} x}{2013} + C$$

(c) $\sec^4 x$

Solution:

$$\int \sec^4 x\, dx = \int \left(\sec^2 x\right)\left(\sec^2 x\right) dx = \int \left(\sec^2 x\right)\left(\tan^2 x + 1\right) dx =$$

$$\int \left(\sec^2 x\right)\left(\tan^2 x\right) dx + \int \sec^2 x\, dx = \frac{\sec^3 x}{3} + \tan x + C$$

(d) $\dfrac{23 - x}{(x-2)(x+5)}$

Solution:

$$\frac{23-x}{(x-2)(x+5)}=\frac{A}{x-2}+\frac{B}{x+5}$$

Solving for A and B, we have A = 3, B = -4.

Hence:

$$\int\frac{23-x}{(x-2)(x+5)}dx=\int\left(\frac{3}{x-2}-\frac{4}{x+5}\right)dx=3\ln|x-2|+4\ln|x+5|+C$$

$(e)\quad \dfrac{x+5}{(x-1)(x^2+5)}$

Solution:

$$\frac{x+5}{(x-1)(x^2+5)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+5}$$

Solving for A, B and C, we have A = 1, B = -1, C = 0

$(f)\quad \dfrac{e^x}{e^{2x}-5e^x+6}$

Solution:

Let u = e^x*; then du =* e^x *dx. So*

$$\int\frac{e^x}{e^{2x}-5e^x+6}dx=\int\frac{1}{u^2-5u+6}du$$

Using partial fraction decomposition:

$$\int \frac{1}{u^2-5u+6}\,du = \int \frac{1}{(u-3)(u-2)}\,du = \int\left(\frac{1}{u-3}-\frac{1}{u-2}\right)du =$$

$$\ln|u-3|-\ln|u-2|+C = \ln|e^x-3|-\ln|e^x-2|+C = \ln\left|\frac{e^x-3}{e^x-2}\right|+C$$

3. [10 pts] (a) Convert $(4,-4\sqrt{3})$ from Cartesian coordinates, (x, y), to polar coordinates, (r, θ). Give an exact answer.

$$r=\sqrt{x^2+y^2}=8 \text{ and } \theta=\arctan\left(-\sqrt{3}\right)=-\frac{\pi}{3}$$

(b) Convert $(8, 2^{2013}\pi)$ from polar coordinates, (r, θ), to Cartesian coordinates, (x, y).

Solution: Note that $2^{2013}\pi$ is a multiple of 2π. Now $x = 8\cos(2^{2013}\pi) = 8\cos 0 = 8$ and $y = 8\sin(2^{2013}\pi) = 8\sin(0) = 0$. Thus the Cartesian coordinates of this point are: ***(8, 0)***

4. [10 pts] Express the polar equation $r(\cos\theta + \sin\theta) = 8$ as an equivalent Cartesian equation.

Solution: Since $x = r\cos\theta$ and $y = r\sin\theta$, we have: $x + y = 8$.

5. [10 pts] Find the *area* bounded by the spiral $r = e^{\theta}$ for $0 \le \theta \le \pi$.

$$A=\int_0^{\pi}\frac{1}{2}r^2\,d\theta=\frac{1}{2}\int_0^{\pi}e^{2\theta}\,d\theta=\frac{e^{2\pi}-1}{4}$$

6 . [10 pts] Find the length of the spiral $r = \theta^2$, $0 \le \theta \le 5^{1/2}$.

$$s = \int_0^{\sqrt{5}} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta = \int_0^{\sqrt{5}} \sqrt{(\theta^2)^2 + (2\theta)^2}\, d\theta = \int_0^{\sqrt{5}} \sqrt{\theta^4 + 4\theta^2}\, d\theta =$$

$$\int_0^{\sqrt{5}} \theta\sqrt{\theta^2 + 4}\, d\theta = \frac{1}{3}\left(9^{3/2} - 4^{3/2}\right) = \frac{19}{3}$$