# Math 162 Solutions: TEST II

**PART I** *(Answer all four problems.)*

1. Compute the value of the following improper integral:

 

*Solution:*



1. Consider the following recursively defined sequence:

c1 = 1,

 for n ≥ 1

(a) Find the values of c2 , c3 and c4.

*Solution:*

*Setting n = 1:*

**

*Setting n = 2:*

**

*Setting n = 3:*

**

(b) Assuming that the limit of cn as *n* → ∞ exists, find its exact value.

*Solution:*

*Assume that L = lim cn exists. Then:*



*and so:*

**

*Multiplying both sides by L yields: L2 = 1 + 3L. So: L2 – 3L – 1 = 0. Using the quadratic formula:*

**

*We reject the negative root, since c1 > 0 and all subsequent terms of the sequence are also positive (reasoning inductively).*

*Thus, if lim cn exists, this limit must be approximately 3.3027.*

1. Determine *convergence* or *divergence* of the following improper integral. Justify your answer:

 

*Solution:*

*Since*  *we have:*



*Thus our integral diverges.*

1. Determine *convergence* or *divergence* of the following improper integral. Justify your answer:



*Solution:*

*Since x3 > x4 when 0 < x <1:*

**

*Now we know that*  *diverges, from the p-test for integrals of the second kind. Thus, invoking the Comparison Test, we find that*  *diverges.*

**PART II** *Select any 5 of the following 6 sequences.* For each selected sequence, determine *convergence* or *divergence*. Briefly justify each answer. *In the case of convergence, find the limit.*  Calculator results will not earn full credit. (You may answer all 6 to earn extra credit.)

1. 

*Solution: Let h = 1/(13n). Then n = 1/(13h) and as n → ∞, h → 0. Hence:*



*Hence the sequence an converges to 1/13.*

2. 

*Solution: Note that:*



*Hence the sequence bn converges to e26.*

3. 

*Rationalizing the “numerator” yields:*

**

*Hence the sequence cn converges to 13.*



*Solution*:





*Solution:*





*Solution:*



**PART III** Select any 5 of the following 6 series. For each selected series, determine *convergence* or *divergence*. Justify each answer. (You may answer all 6 to earn extra credit.)

1. 

*Solution: This series is telescoping. Consider the sequence of partial sums:*



*from which we infer that sn = 1/13 – 1/(13+n) → 0 as n → ∞. Thus the sum of our series is 1/13.*

2. 

*Solution: Applying the ratio test*

**

*we find that the series converges since r < 1.*

3. 

*Solution: Since  we apply the nth Term Test for Divergence to conclude that our series diverges.*

4. 13.13131313…

*Solution: Observe that 13.13131313… = 13(1 + 10-2 + 10-4 + 10-6 + …).*

*Ignoring the factor of 13 for the moment, we have a geometric series with ratio R = 0.01. Hence* 

1. 

*Solution: Using the Comparison Test:*



*Using the p-test for the larger series, we see that our series converges.*

1. 

*Solution: Consider the following inequality:*



*Using the p-test, we see that the smaller series diverges and hence our series diverges as well.*

**PART IV** Select any five of the following six problems. You may answer all six for extra credit. For each improper integral below, determine convergence or divergence. *Justify each answer!*



*Solution:Since*



*we see that*



*Hence our integral diverges.*



*Solution:*

*Since ln x < x , we have:*



*Thus, invoking both the p-test and the Comparison Test, our original integral converges.*



*Solution:*

*Observe that*





*Thus, invoking the Comparison Test, our original integral converges.*



 *Solution:*

*Observe that*



*Thus, invoking the Comparison Test, our original integral converges.*



*Solution:*

*Observe that*



*Thus, invoking the Comparison Test, our original integral diverges.*

 (*Hint:* Use a common identity.)

*Solution:*



*Thus the original integral diverges.*

**PART V** Select any 4 of the following 5 problems. You may answer all five for extra credit. For each numerical series below, determine *convergence* or *divergence*. Justify each answer.

1. 

*Solution: Applying the ratio test to this positive series:*

**

*Since r < 1, we conclude that our positive series converges.*

2. 

*Solution: Applying the nth root test to this positive series:*



*Since  < 1, we conclude that our positive series converges.*

3. 

*Solution: Applying the ratio test to this positive series:*

**

4. 

 *Solution: Since*



*We may invoke the n th Term Test for Divergence to conclude that our original series diverges.*

5. 

*Solution:*

*Since  we can invoke the Comparison Test to conclude that our series diverges.*