Solutions: TEST III

**Instructions:** *Answer any 9 of the following 11 problems. You may answer more than 9 to obtain extra credit.*

1. Without using l'Hôpital’s rule, find:



*Solution:*

*Since*



*it follows that:*



*Since*



*it follows that:*



*Hence:*



1. Evaluate each of the following indefinite integrals.
2. 

*Solution:*

*After completing the square, make the change of variable u = x + 5 (and thus du = dx):*

**

1. 

*Solution:*

*Let x = tan t. Then dx = sec2 t dt. Hence:*



1. Evaluate each of the following indefinite integrals. Show your work.
2. 

*Solution:*



(b) 

*Solution: Since sin(4x) = sin(3x+x) = sin 3x cos x + cos 3x sin x,*

*we obtain: sin 3x cox x = ½ (sin 4x + sin 2x). Thus:*



(c) 

*Solution:*



*Introduce the change of variable u = sin x (and thus du = cos x dx) to obtain:*



1. Given y = G(x) below, calculate the value of G(1313)(0). (Express your answer in factorial form.)



*Solution:*

*Beginning with the Maclaurin series for sinh t and then replacing t by x2:*



*Now, multiplying by x3 yields:*



*Now, the general Maclaurin series of G(x) is:*

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*Thus the coefficient of x1313 is G(1313)(0) / 1313!*

*Now the series for x3 sinh (x2) has coefficient of x1313 occur when 4n + 5=1313, that is, when n =327 (and so 2n + 1=655). Thus this coefficient is: 1 / 655!*

*Equating G(1313)(0) / 1313! with 1 /655!, we find that:*

***G(1313)(0) = 1313! / 655!***

**5.** (a) Using Euler’s formula, find a formula for sin(4x) in terms of sin x and cos x.

*Solution:*

Since exi = cos x + i sin x, we have:

e4xi = (cos x + i sin x)4 = (cos x)4 + 4(cos x)3 i sin x + 6 (cos x)2 i2 (sin x)2 + 4 (cos x) i3 (sin x)3 + (i4) (sin x)4 = (cos x)4 + 4 (cos x)3 (sin x) i– 6 (cos x)2(sin x)2 – 4 (cos x)(sin x)3 i + (sin x)4

But Euler’s formula tells us that e4xi = cos(4x) + i sin(4x).

Thus **sin(4x) = 4 cos3 x sin x – 4 cos x sin3 x**

(b) Using Euler’s formula find all the solutions of the equation

*z4  = – i*

Express each answer in the form a + bi.

*Solution:*

*Since*



*we find that the four fourth roots of –i are:*



 **6**. By dividing power series, find the *first four non-zero* terms of the Maclaurin series of



*Solution:*



**7.** Through an appropriate change of variables, convert each of the following to a trigonometric integral. *Do not evaluate.*



*Solution:*

*Let x = sec . Then* ***dx = sec tan  d*** *and so:*





*Solution:*

*Let x = 3 tan . Then* ***dx = 3 sec2 d*** *and so:*



**8.** For each pair of integrals, determine which one is the *more difficult* to evaluate. (You need not evaluate any of these integrals.) *Briefly explain!*



*The first integral is easier to evaluate since it is of the form*



*The second integral requires that one begin with the double angle formulas for sine and cosine.*



*The first integral is easier to evaluate since it is of the form*



**9.**For each series below, determine *absolute convergence*, *conditional convergence* or *divergence*. Justify each answer.



*Solution:*

*Notice that this series fails to converge absolutely, by the p-test. It does converge, however, due to Cauchy’s test. Thus the series converges conditionally.*



*Solution:*

*Since arctan(k2) → /2 as k → ∞, the series diverges by the nth-term test for divergence.*



*Applying the Ratio Test, we see that the series converges absolutely:*



**10.** For each power series below, determine the *interval of convergence*. Do not investigate the behavior of each power series at the endpoints.

 

*Using the ratio test:*



*Thus the series converges absolutely for |x – 13| / 13 < 1. So the interval of convergence is (0, 26).*

 (b) 

*Invoking the root test:*



*Thus the series converges absolutely for e |x – 4| < 1. So the interval of convergence is (4 – 1/e, 4 + 1/e).*

**11.** For the power series below, determine the *interval of convergence*. Investigate *end point behavior*.



*Solution:*

*Using the ratio test:*



*Thus the series converges absolutely for |x – 13| < 1. So the interval of convergence is (12, 14).*

*At x = 14, the series equals:*

**

*which diverges (using the comparison test and the p-test).*

*At x = 12, the series equals:*

**

*which converges conditionally (using Cauchy’s test as well as the fact that it fails to converge absolutely).*

 (b) 

*Solution:*

*Using the ratio test:*



*Thus the series converges absolutely for 13x2 < 1. So the interval of convergence is* 





*which converges absolutely (using the p-test).*





*which converges absolutely (using the p-test).*

***Extra Credit:*** Using a series representation of sin(3x), find constants

*r* and *s* for which:



*Solution:*

*Since*



*we have:*



*If this limit equals 0, then 3 + r = 0 and* s – 33/(3!) = 0.

*Hence r = -3 and s = 9/2.*