WORKSHEET XV

OPERATIONS ON POWER SERIES

1. (a) Consider the power series $f(x) = 1/(1 - x) = 1 + x + x^2 + x^3 + \ldots$ What is its interval of convergence?

   (b) Find the series for $f'(x)$ *and* $f''(x)$.

2. Using the power series obtained in 1(b) for $f'(x)$, determine the sum of the series $\sum_{n=1}^{\infty} \frac{n}{3^n}$

3. The series

   $\sin x = x - x^3/3! + x^5/5! - x^7/7! + x^9/9! - x^{11}/11! + \ldots$

   converges to sin x for all real x.

   (a) Find the first six terms of a series for cos x. For which values should the series converge?

   (b) By replacing x by 2x in the series for sin x, find a series that converges to sin 2x for all *x*.

   (c) Using series multiplication, find a series that converges to 2 sin x cos x.

4. (a) On which interval does the series $1/(1 + t) = 1 - t + t^2 - t^3 + t^4 - t^5 + \ldots$ converge?

   (b) Integrating both sides over the interval [0, x], find the first six terms of a series that converges to ln(1 + x).

5. The series $e^x = 1 + x + x^2/2! + x^3/3! + x^4/4! + x^5/5! + \ldots$ converges to e^x for all real *x*.

   (a) Find a series for $(d/dx)\, e^x$.

   (b) Find a series for $\int e^x\, dx$

   (c) In the series for e^x, replace x by –x to find a series expansion of e^{-x}.

   (d) In (c), replace x by x^2 to find a series expansion of e^{-x^2}.

6. (a) Beginning with the series for $1/(1 + x^2)$, find a series expansion of arctan x.

(b) Find a series expansion for $\int \frac{\arctan x}{x}\, dx$.

7. Using a series representation for sin 3x, find values of *r* and *s* for which

$$\lim_{x\to 0}\left(\frac{\sin 3x}{x^3}+\frac{r}{x^2}+s\right)=0$$

I used to love mathematics for its own sake, and I still do, because it allows for no hypocrisy and no vagueness….

- Stendhal, **The Life of Henri Brulard**