## WORKSHEET II

## VOLUME \& CAVALLERI'S PRINCIPLE



## WOLFRAM DEMO

DEMO 2

1. The base of a solid is the region bounded by the curve $y=(\sin x)^{1 / 2}$ and the interval $[0, \pi]$ on the x -axis. The cross sections perpendicular to the x axis are equilateral triangles with bases running from the x -axis to the curve. Find the volume of this solid.
2. The cross sections of a solid are squares perpendicular to the x -axis with their centers on the axis. If the square cut off at $x$ has edge length of $2 x^{2}$, find the volume of the solid between $\mathrm{x}=0$ and $\mathrm{x}=\mathrm{a}$.
3. Find the volume of a right circular cone of height $h$ and base radius $r$.
(Hint: Revolve an appropriate triangle about the x or y -axis.)
4. Consider the triangle $T$ with vertices $(0,0),(2,0)$, and $(1,1)$. Find the volume of the solid of revolution obtained by rotating $T$ about:
(a) the x -axis
(b) the $y$-axis
(c) the vertical line $\mathrm{x}=3$
(d) the horizontal line $y=-1$
(e) The horizontal line $\mathrm{y}=2$
5. Consider the portion of the ball of radius $R$ centered at the origin for $y \geq$ $R-h$ where $0<h<R$. Find the volume of this spherical cap.

6. Find the volume of the torus obtained by revolving the disk $\mathrm{x}^{2}+\mathrm{y}^{2} \leq \mathrm{a}^{2}$ about the line $\mathrm{x}=\mathrm{b}$, where $\mathrm{b}>\mathrm{a}$.

7. Consider the region $R$ bounded by the curves $y=x^{2}$ and $y=2-x$. Find the volume of the solid obtained by rotating $R$ about axis $x=-3$.
8. Let C be the region bounded by the lines $\mathrm{y}=\mathrm{x}, \mathrm{y}=2 \mathrm{x}$ and $\mathrm{y}=2$. Find the volume of the solid obtained by rotating $C$ about the $x$-axis.
9. Consider the region $A$ bounded by the curve $y=x^{2}-x^{3}$ and the $x$-axis.

Find the volume obtained by rotating about:
(a) the $y$-axis
(b) the vertical line $\mathrm{x}=1$
(c) the vertical line $\mathrm{x}=3$
(d) the vertical line $x=-3$
10. The region in the $x y$-plane defined by the inequalities $0 \leq x \leq 2$ and $x^{2} / 4$
$\leq y \leq 1$ is rotated about the given axis below. Find the volume of the solid of revolution so generated.
(a) the x -axis
(b) the $y$-axis
(c) the vertical line $\mathrm{x}=2$
(d) the horizontal line $\mathrm{y}=1$


Bonaventura Francesco Cavalieri (1598-1647)

