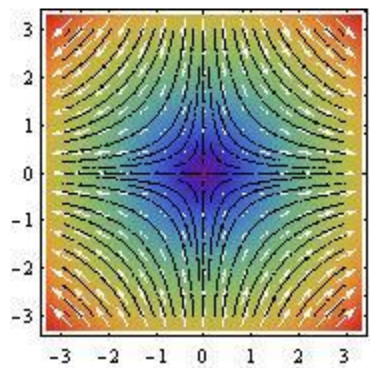
WORKSHEET XXII

SEPARABLE DIFFERENTIAL EQUATIONS,

1ST ORDER LINEAR DIFFERENTIAL EQUATIONS,

AND DIRECTION FIELDS



I Solve each of the following separable differential equations.

1. $dy/dx = y(1 - y)$

2. $x\, dy/dx = 2(y - 4)$

3. $\dfrac{dy}{dx} = \dfrac{x^2 y - 4y}{x + 2}$

4. $\dfrac{dy}{dx} = xe^{x^2 - \ln(y^2)}$

5. $dy/dx = (x+y)/x$ (*Hint:* Let $v = y/x$.)

6. $\dfrac{d}{dx}\left(xe^x y\right) = 2e^{2x}$

7. $(1 + x^2)\, dy/dx = \arctan x$

8. $\frac{dy}{dx} = \frac{y-1}{xy}$

9. $xyy' = (x+a)(y+b)$ where *a* and *b* are constants.

10. $x\frac{dy}{dx} = (1-2x^2)\tan y$

11. $L\frac{di}{dt} + Ri = 0$ where *L* and *R* are constants.

12. $x\frac{dy}{dx} = y + x^3 \tan\frac{y}{x}$ (*Hint:* Let v = y/x.)

13. dP/dt = cP(1 – P/K) (*the logistic equation*)

II Solve each of the following initial value problems.

1. $dy/dx = 6y^2x$, $y(1) = 1/25$

2. $\frac{dy}{dx} = \frac{3x^2+4x-4}{2y-4}, \quad y(1) = 3$

3. $\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}, \quad y(0) = 1$

4. $dy/dx = e^y(2x - 4)$, $y(5) = 0$

5. $e^{\frac{dy}{dx}} = x^4, \quad y(1) = 1$

III By selecting an appropriate integrating factor, solve each of the following first-order linear differential equations:

1. $x(dy/x) + y = x^3, x > 0$
2. $x(dy/dx) + y = e^x, x > 0$
3. $e^x(dy/dx) + 2 e^x y = 1$
4. $x(dy/dx) + 3y = (\sin x)/x^2, x > 0$
5. $dy/dx + (\tan x) y = \cos^2 x, -\pi/2 < x < \pi/2$
6. $x(dy/dx) + 2y = 1 – 1/x, x > 0$
7. $(t – 1)^3(ds/dt) + 4(t – 1)^2 s = t + 1, t > 1$

IV Draw *isoclines* and *direction fields* for each of the following equations. Sketch some of the *integral curves*.

1. $dy/dx = x^2 – y$
2. $dy/dx = – x^2$
3. $dy/dx = x – 2y$
4. $dy/dx = 1/y$
5. $dy/dx = xy$

Often there is little resemblance between a differential equation and its solution. Who would suppose that an expression as simple as $dy/dx = 1/(a^2-x^2)$ could be transformed into $y = (1/2a) \ln (a+x)/(a-x) + C$? This resembles the transformation of a chrysalis into a butterfly!

- Silvanus P. Thompson