

WORKSHEET VIII

LITTLE OH AND BIG OH

Suppose that $f(x) \rightarrow \infty$ and $g(x) \rightarrow \infty$ as $x \rightarrow \infty$. We say that “ f is of smaller order than g ” if $\frac{f(x)}{g(x)} \rightarrow 0$ as $x \rightarrow \infty$. In this case we write $f = o(g)$.

Assume that f and g are each positive for large x . We say that “ f is at most the order of g ” if there is a positive integer M for which $\frac{f(x)}{g(x)} \leq M$ for large x . In this case we write $f = O(g)$.

Determine which of the following statements are true; justify each answer.

(a) $3x^2 + 11 = o(x^5 + x + 99)$

(b) $x + 5 \sin x = O(x)$

(c) $2^x = o(x^{100})$

(d) $3^x = O(e^x)$

(e) $x = o(\ln x)$

(f) $3 + \ln x + \ln(\ln x) + \sqrt{x} = o\left(x^{\frac{2}{3}}\right)$

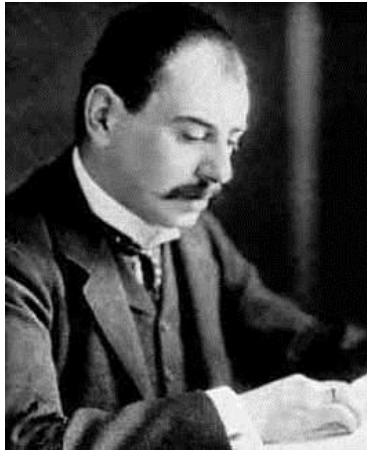
(g) $\ln x = o(\sqrt{x})$

(h) $(x^2+1)^4 = O((2x+1)^3x^5)$

(i) $\frac{x^2 + 13x + 2009}{5x + 1789} = O(\sqrt{x^2 + 9})$

(j) $\ln x = o(\ln(\ln x))$

(k) $\ln(x^{55} + x^{33} + x^{11} + 1) = O(\ln x)$



[Edmund Landau](#) (1877 – 1938) is known for his work in analytic number theory and the distribution of primes. He first introduced the *little oh* notation in 1909.

[COURSE HOME PAGE](#)

[DEPARTMENT HOME PAGE](#)

[LOYOLA HOME PAGE](#)