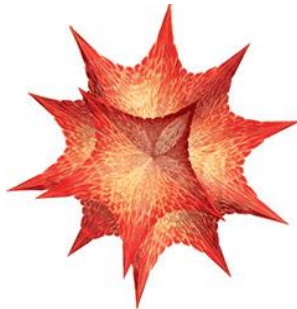


MATHEMATICA LAB III



SEQUENCES

(Lab report due: 25 March 2015)

(A) graphing sequences

1. Using the *ListPlot* command in Mathematica, plot the ordered pairs (n, a_n) for the first 100 terms of each of the seven sequences below. Based upon this evidence, can you guess whether the sequence converges or diverges? In the former case, try to *estimate* the limit of the sequence. Then find the *exact value* of the limit using the *Limit* command.

$$(a) \quad a_n = \frac{\ln n}{\sqrt{n}}$$

$$(b) \quad b_n = \frac{n^{10}}{2^n}$$

$$(c) \quad c_n = \frac{10^n}{n!}$$

$$(d) \quad d_n = \frac{(3n+1)^3(2n-4)^5}{(2n+3)^4(n+11)^4} \quad \text{Note: Here you will need to examine at least a few thousand terms.}$$

(e) $e_n = n^{\frac{1}{n}}$

(f) $f_n = \ln(3n^3 - n^2 + 5) - \ln(n^3 + 5n + 89)$

(g) $g_n = \sqrt{n^4 + 4n^2 + 5n + 13} - \sqrt{n^4 - n^2 + 9n + 5}$

2. (a) It is a well-known and important fact that the sequence $\left\{ \left(1 + \frac{r}{n}\right)^n \right\}$ converges to e^r . By letting $r = 1$, $\ln 2$, and $\ln 3$, support this result using Mathematica.

- (b) Investigate the behavior of the two related sequences $\left\{ \left(1 + \frac{1}{n^2}\right)^n \right\}$ and

$\left\{ \left(1 + \frac{1}{\sqrt{n}}\right)^n \right\}$. Discuss how the sequences differ in their behavior as n grows

large. What do you think accounts for these differences?

3. Investigate the behavior of the sequence $\left\{ (n!)^{\frac{1}{n}} \right\}$. Using Stirling's formula, find a simpler expression that approximates this sequence.

(B) recursively defined sequences

Using RecurrenceTable, ListPlot and the Nest command, analyze each of the following sequences and find the limit if it exists.

4. $c_1 = 2$, and $c_n = (c_{n-1} + 3/c_{n-1})/2$ for $n \geq 2$.
5. $c_1 = 2$, and $c_n = (c_{n-1} + 5/c_{n-1})/2$ for $n \geq 2$.
6. $x_1 = 1$ and $x_n = x_{n-1} + \cos(x_{n-1})$ for $n \geq 2$. Can you find the exact value of this limit without employing Mathematica?

7. $y_1 = 5$ and $y_n = 4/(3 + 2y_{n-1})$ for $n \geq 2$.
8. $z_1 = 4$ and $z_n = \cos(z_{n-1})$ for $n \geq 2$.
9. $q_1 = 3$, $q_n = 2 + 1/\sin(q_{n-1})$ for $n \geq 2$.



Recursive fish by Dan Goodman, Math Dept, University of Warwick