MATHEMATICA LAB IV

TAYLOR SERIES

(Due: 20 April 2015)

I Power Series

1. Find the 8^{th} order Maclaurin polynomial of tanh(x).
2. Find the 7^{th} order Taylor polynomial of e^x about x = 1.
3. Find the 9^{th} order Maclaurin polynomial of ln(1+x).
4. Plot the graph of y = e^x along with the first four Maclaurin polynomials of e^x on the same set of axes.
5. Find the 14^{th} order Maclaurin polynomial of exp(x^2). Can you see how this polynomial is related to the 7^{th} order Maclaurin polynomial of e^x ? Explain.

II Weierstrass' example

Here we examine a function defined by an infinite series (that is not a power series) which is continuous but nowhere differentiable.

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{2^n} \sin\left(3^n x\right)$$

6. Plot the n^{th} partial sum of f(x) for several values of n (for example, n = 3, 5, 8). Why might you believe that f(x) is not differentiable?

III Infinite products

In mathematics, infinite products play an important role, although perhaps not quite as important a role as that of infinite series. Analogous to infinite series, an infinite product is the limit of a sequence of partial products. The capital Greek letter, pi, is used to indicate a product. For example,

$$\prod_{k=1}^{n} a(k)$$

denotes the product: a(1)a(2)a(3)...a(n). If we wish to define an *infinite product*, we could let p(n) = a(1)a(2)a(3)...a(n) and define the infinite product to equal the limit of p(n) as n increases without bound, if the limit exists. Of course, if the limit does not exist, we say that the infinite product diverges.

7. Examine the infinite products defined by a(n) = 1 + 1/n and b(n) = 1 + $1/n^2$. Graph each sequence of partial products. Does either infinite product converge? If so, what is its limit?

Brook Taylor (1685 – 1731)