MATH 162 PRACTICE QUIZ V

1. For each improper integral (of the second kind) below, determine convergence or divergence. For those that converge, compute its value.

$$(a) \quad \int_{0+}^{1} \frac{1}{\sqrt{x}}\, dx$$

$$(b) \quad \int_{0}^{1-} \frac{1}{\sqrt{1-x^2}}\, dx$$

$$(c) \quad \int_{0}^{\frac{\pi}{2}-} \sec x\, dx$$

$$(d) \quad \int_{0+}^{e} \ln t\, dt$$

2. For which value(s) of the constant C will the following improper integral converge?

$$\int_{4}^{\infty} \left(\frac{2x}{x^2+1} - \frac{C}{2x+1} \right) dx$$

3. Find the *volume* of the solid of revolution obtained by rotating the curve $y = 1/x^2$ from $x = 1$ to $x = \infty$ about the x-axis or explain why no such number exists.

4. For each of the following improper integrals, determine convergence or divergence. Use an appropriate version of the Comparison Test.

$$(a) \quad \int_{0+}^{\infty} \frac{1}{x^{\frac{2}{3}} + x^{\frac{4}{3}}}\, dx$$

(b) $$\int_{0+}^{\infty} \frac{1+x}{x^3+\sqrt{x}}\,dx$$

(c) $$\int_{0}^{\frac{\pi}{2}-} \tan x\,dx$$

(d) $$\int_{0+}^{1} \frac{1-\ln x}{x^4}\,dx$$

(e) $$\int_{0}^{\frac{\pi}{2}-} \tan x\,dx$$

(f) $$\int_{0}^{2-} \frac{1}{\sqrt{4-x^2}}\,dx$$

(g) $$\int_{0+}^{1} \frac{1-\ln x}{x^4}\,dx$$

5. For each of the following sequences, determine *convergence* or *divergence*. In the case of convergence, find the *limit* of the sequence. *Briefly explain your reasoning!*

(a) $$c_n = \sqrt{1-\frac{3}{n}}$$

(b) $$d_n = \frac{\cos^4(n^3+n^2+5)}{n^2+23}$$

(c) $e_n = 1 + \arctan n$

(d) $u_n = \dfrac{e^n}{n^5}$

(e) $z_n = \dfrac{(n^5+1)^3(1-4n)^2}{(n+9)^{17}}$

6. Consider the following *recursively defined* sequence:

$a_1 = 4$

$a_2 = 2$

$a_n = a_{n-1}a_{n-2} - a_{n-1} - a_{n-2} + 1$ for $n \geq 3$.

Find the numerical values of a_3, a_4, a_5 and a_6. (Show your work.)

7. To which of the following series does the "*n^{th} term test for divergence*" apply? Explain!

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+5}$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$

$$(e) \quad \sum_{n=1}^{\infty} \arctan(n)$$

$$(f) \quad \sum_{n=1}^{\infty} n^{1/n}$$

8. For n ≥ 1, let

$$a_n = \int_0^1 (x^2+2)^n \, dx.$$

Determine convergence or divergence of the sequence $\{a_n\}$. (*Hint:* Do *not* try to evaluate the integral! Calculator solutions are not accepted.)

Hint: Is the sequence *monotone*?

9. Let $a_n = 1/1 + ½ + 1/3 + ¼ + \ldots +1/n$ for n ≥ 1(integers only)

Demonstrate that the sequence $\{a_n\}$ diverges.

10. Assuming that the limit exists, find it.

$$a_1 = 1+\sqrt{2},\ a_2 = 1+\sqrt{1+\sqrt{2}},\ a_3 = 1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{2}}}}, \ldots$$

11. By computing the first few terms, guess what the limit of the following recursively defined sequence.

$$a_1 = 1,\ a_n = \frac{1}{2}\left(a_{n-1} + \frac{5}{a_{n-1}}\right) \ \textit{for}\ n \ge 2$$

There is more danger of numerical sequences continued indefinitely than of trees growing up to heaven. Each will some time reach its greatest height.

- Friedrich Ludwig Gottlob Frege, **Grundgesetz der Arithmetik** (1893)