MATH 162 PRACTICE QUIZ VI 25 FEBRUARY 2015

Name: ________________________

1. To which of the following series does the "*n^{th} term test for divergence*" apply? Explain!

$$(a)\quad \sum_{n=1}^{\infty} \frac{n}{n+5}$$

$$(b)\quad \sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n$$

$$(c)\quad \sum_{n=1}^{\infty} \frac{1}{n}$$

$$(d)\quad \sum_{n=1}^{\infty} \frac{1}{n^3}$$

$$(e)\quad \sum_{n=1}^{\infty} \arctan(n)$$

$$(f)\quad \sum_{n=1}^{\infty} n^{1/n}$$

2. Which of the following infinite series are *geometric* and which are not? For those that are geometric, determine convergence or divergence. In the case of convergence, find the limit. Show your work!

$$(a)\quad \sum_{n=0}^{\infty} \frac{5^{n+1}}{9^{n-1}}$$

$$(b)\quad \sum_{n=0}^{\infty} \frac{3^n 2^{2n+1}}{5^n}$$

$$(c)\quad \sum_{n=0}^{\infty} \frac{2^n}{n^n}$$

$$(d)\quad \sum_{n=0}^{\infty} \frac{13(2^{3n+7})}{9^n}$$

$(e)\ \sum_{n=0}^{\infty} \frac{3^n+1}{4^n}$

3. Sum each of the following series or explain why the series is divergent.

$(a)\ \sum_{n=3}^{\infty} \frac{1}{n(n+1)}$

$(b)\ \sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$

$(c)\ 0.17891789178 9...$

$(d)\ 5.1343434343434...$

4. For each of the following positive series determine convergence/ divergence of the series. You may need to use the comparison test. Justify your answers!

(a) $\sum_{n=1}^{\infty} \cos\left(\frac{1}{5+n^7 \ln n}\right)$

(b) $\sum_{n=2}^{\infty} \frac{1+n^2+n^4}{19+n+n^4\sqrt{n}}$

(d) $\sum_{n=1}^{\infty} \left(\frac{1+45\ln n+n}{5+n^2}\right)^2$

(e) $\sum_{k=1}^{\infty} \frac{k(k+1)(k^2+5)}{(k-13\ln k)^4}$

(f) $$\sum_{n=1}^{\infty} \frac{1+13n+2^n}{1+2015n+e^n}$$

5. *True or False:*

(a) An increasing sequence bounded above must converge.

(b) A convergent sequence is bounded.

(c) A convergent sequence is either increasing or decreasing.

(d) A decreasing sequence bounded below must converge.

(e) A positive series must converge.

(f) A positive series is either convergent or unbounded.

(g) The harmonic series diverges.

(h) Consider the two series $\Sigma\, a_n$ and $\Sigma\, b_n$ where the summation begins at n = 0.

If Σa_n and Σb_n each converge to 1 then $\Sigma\, (3a_n + b_n)$ must converge to 4.

Divergent series are the invention of the devil, and it is shameful to base on them any demonstration whatsoever.

- Niels Henrik Abel (1828)

There is more danger of numerical sequences continued indefinitely than of trees growing up to heaven. Each will some time reach its greatest height.

- Friedrich Ludwig Gottlob Frege, **Grundgesetz der Arithmetik** (1893)