

1. For each numerical series below, determine *convergence* or *divergence*. In the case of convergence, determine if the series converges *absolutely* or *conditionally*. Justify each answer.

$$(a) \sum_{m=1}^{\infty} \left( \frac{-e}{m} \right)^m$$

$$(b) \sum_{n=1}^{\infty} \frac{(-1)^{3n} 2^n}{\left( 1 + \frac{1}{n} \right)^{n^2}}$$

$$(c) \sum_{n=3}^{\infty} (-1)^n \frac{5}{\ln n}$$

$$(d) \sum_{n=1}^{\infty} \frac{(-1)^n (n!)^2 3^n}{(2n+1)!}$$

$$(e) \sum_{k=1}^{\infty} (-1)^k \frac{k(k+1)(k^2+5)}{(k-13 \ln k)^4}$$

$$(f) \sum_{n=1}^{\infty} \frac{5^n + 7}{11^n}$$

$$(g) \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (3n)!}{(n!)^3}$$

$$(h) \sum_{n=1}^{\infty} (-1)^n \frac{4^n (n!)^2}{(2n)!}$$

2. For each of the following numerical series, determine if the series *diverges*, *converges conditionally* or *converges absolutely*. Justify your answers!

$$(a) \sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{1}{5 + n^7 \ln n}\right)$$

$$(b) \sum_{n=2}^{\infty} \frac{\sin(3n + 5)}{n(\ln n)^2}$$

$$(c) \sum_{n=3}^{\infty} (-1)^n \frac{5}{\ln n}$$

$$(d) \sum_{n=1}^{\infty} (-1)^n \left(\frac{1+n}{5+n^2}\right)^n$$

$$(e) \sum_{k=1}^{\infty} (-1)^k \frac{k(k+1)(k^2+5)}{(k-13 \ln k)^4}$$

$$(f) \quad \sum_{n=1}^{\infty} (-1)^n \frac{2^n}{\left(1 + \frac{1.3}{n}\right)^{n^2}}$$

3. Does absolute convergence imply convergence? Does convergence imply absolute convergence? Why?

4. How many terms are required in order to estimate each of the following sums accurately to 4 decimal places?

$$(a) \quad \sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2}$$

$$(b) \quad \sum_{n=1}^{\infty} (-1)^n \frac{1}{n^8}$$

$$(c) \quad \sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln n}$$

5. For each of the following *power series*, determine the *interval of convergence*. Do not study end-point behavior.

$$(a) \quad \sum_{n=1}^{\infty} \frac{3^n}{n(n+5)} x^n$$

$$(b) \quad \sum_{n=1}^{\infty} \frac{1}{n(n+3)(n+11)} (x-4)^n$$

$$(c) \quad \sum_{n=1}^{\infty} \frac{n^n}{n!} (x-1)^n$$

$$(d) \quad \sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} (x-4)^n$$

$$(e) \quad \sum_{n=0}^{\infty} \frac{1}{2^n \sqrt{n+1}} (x+3)^n$$

$$(f) \quad \sum_{n=1}^{\infty} \frac{7^n}{n^5 e^n} x^n$$

*Pure mathematics is, in its way, the poetry of logical ideas.*

- Albert Einstein