MATH 162 PRACTICE QUIZ VIII

1. For each of the following functions, find the 4^{th} order Taylor polynomial centered at $x = c$:

   (a) $y = \sinh x + 3 \cosh x$, $c = 0$

   (b) $y = 1 + x + e^{3x}$, $c = 0$

   (c) $y = 1/(x + 2)$, $c = 0$

   (d) $y = \ln(1 + x)$, $c = 0$

   (e) $y = x^{1/2}$, $c = 4$

   (f) $y = \sin x$, $c = \pi/4$

   (g) $y = 1 + x + 3x^2 - 4x^3$, $c = 0$

   (h) $y = 1 + x + 3x^2 - 4x^3$, $c = 1$

   (i) $y = xe^{2x}$, $c = 0$

2. Using multiplication of power series, find the *first four non-zero* terms of the Maclaurin series expansion of $f(x) = e^{2x} \cos(3x)$.

3. Using division of power series, find the *first four non-zero* terms of the Maclaurin series expansion of

$$G(x) = \frac{\cosh x}{1 + x + x^2}$$

4. Using your choice of technique, find the *first four non-zero* terms of the Maclaurin series expansion of:

   (a) $y = xe^{-4x}$

   (b) $y = (2 + x)/(1 - x)$

   (c) $y = (1 - x - x^2)\, e^{2x}$

(d) $y = (\sin x) \ln(1 + x)$

(e) $y = x \cos^2 x$

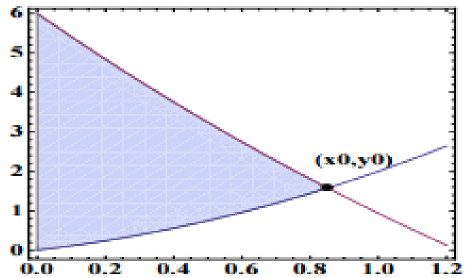
(f) $y = e^{x^3}$

(g) $y = \exp(1 + x^2)$

5. Find the Taylor series expansion of $y = e^x$ at $x = c$.

6. *(University of Michigan final exam problem)*

[15 points] The graph shows the area between the graphs of $f(x) = 6\cos(\sqrt{2x})$ and $g(x) = x^2 + x$. Let (x_0, y_0) be the intersection point between the graphs of $f(x)$ and $g(x)$.



a. [6 points] Compute $P(x)$, the function containing the first three nonzero terms of the Taylor series about $x = 0$ of $f(x) = 6\cos(\sqrt{2x})$.

b. [3 points] Use $P(x)$ to approximate the value of x_0.

c. [3 points] Use $P(x)$ and the value of x_0 you computed in the previous question to write an integral that approximates the value of the shaded area. Find the value of this integral.

d. [1 point] Graph $f(x)$ and $g(x)$ in your calculator. Use the graphs to find an approximate value for x_0.

e. [2 points] Write a definite integral in terms of $f(x)$ and $g(x)$ that represents the value of the shaded area. Find its value using your calculator.

7. Without using L'Hôpital's rule, find

$$\lim_{x \to 0} \frac{\ln(1+x) - x}{1 - \cos x}$$

8. By differentiating an appropriate power series, compute the following sum:

$$\sum_{n=1}^{\infty} \frac{n}{5^n}$$

9. Find the Taylor series of

$$F(x) = \cos\sqrt{x+1}$$

centered at x = -1.

10. Let $F(x) = x^4 \arctan(3x)$. Find $F^{(2345)}(0)$.

Hint: Beginning with a geometric series, find the Maclaurin series expansion of arctan(t).

11. Without using L'Hôpital's rule, find

$$\lim_{x \to 0} \frac{e^{2x} - 4\cos\left(x^2\right) - 2x + 5 - 2e^{x^2}}{\sin(x^3) + x^5 e^x}$$

12. Find the first four non-zero terms in the Maclaurin expansion of f(x) = tan x by dividing the series for sin x by the series for cos x.

One cannot escape the feeling that these mathematical formulas have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers.

- Heinrich Hertz