

# MATH 162

# PRACTICE QUIZ IX

1. Integrate each of the following functions:

(a)  $\tan^5 x$

(b)  $\sec^3 x \tan^5 x$

(c)  $\tan^6 2x$

(d)  $\sec^4 x$

(e)  $(\sec x)^{-4/3} \tan x$

(f)  $(\sin 10x)(\sin 5x)$

(g)  $\sin^9 x \cos^{11} x$

(h)  $(\cos 4x)(\sin 3x)$

2. Integrate each of the following functions:

(a)  $\frac{x^7 + 1}{x - 1}$

(b)  $\frac{x^2 + 2}{x^2 - 9}$

(c)  $\frac{x - 3}{(x - 1)(x - 2)^2}$

$$(d) \frac{1}{(x^2 + 1)(x - 2)^2}$$

$$(e) \frac{e^x}{e^{2x} - 5e^x + 4}$$

$$(f) \frac{\cos x}{2 \sin^2 x + 7 \sin x + 6}$$

$$(g) \frac{\sinh x}{(\cosh x)^2 - \cosh x}$$

3. State *Euler's identity*. Use Euler's identity to express  $\cos(4x)$  in terms of  $\cos x$  and  $\sin x$ .

4. Solve the equation  $z^4 = -16$ .

5. Express each of the following in the form  $a + bi$

(a)  $i^{-1}$

(b)  $(-1)^i$

(c)  $(1 + i)^{90}$

(d)  $3e^{\pi i/6}$

(e)  $(\sqrt{3} + i)^{11}$

(f)  $i^{i^i}$

6. Using substitution (or any other method that you prefer), evaluate each of the following integrals:

$$(a) \int \frac{\tan(\ln x)}{x} dx$$

$$(b) \int \sqrt{x} \sin(2x^{3/2}) dx$$

$$(c) \int e^x \sec^2(e^x - 13) dx$$

$$(d) \int \sec x \tan x \sqrt{1 + \sec x} dx$$

$$(e) \int \sqrt{a + b\sqrt{c + x}} dx$$

$$(f) \int \frac{\arcsin \sqrt{x}}{2\sqrt{x - x^2}} dx$$

$$(g) \int \frac{dx}{\sqrt{\arctan x} (1 + x^2)}$$

7. Express each of the following in *polar form*: (a)  $1 + i$ , (b)  $3 - 3i$ ,

(c)  $4 - 4\sqrt{3}i$ , (d)  $5 + 12i$

8. Solve the equation  $z^5 = 1$ . (You should have five solutions.)

9. Solve the equation  $z^4 = -1$ .

10. Solve the equation  $z^3 = i$ .

11. Using Euler's formula, express  $\sin 5x$  in terms of  $\sin x$  and  $\cos x$ .

(Hint:  $(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$ )

12. Using an appropriate trig substitution, evaluate OR reduce each of the following to a simpler form:

(a)  $\int \frac{1}{4 + z^2} dz$

(b)  $\int \frac{1}{\sqrt{9 - x^2}} dx$

(c)  $\int \frac{x^2}{\sqrt{25 - x^2}} dx$

(d)  $\int \sqrt{25t^2 - 36} dt$

(e)  $\int \frac{\sqrt{y^2 - 49}}{y} dy$  for  $y > 7$

(f)  $\int \frac{\sqrt{w^2 - 9}}{w^3} dw$  for  $w > 3$

(g)  $\int \frac{1}{v^2 \sqrt{v^2 - 1}} dv$  for  $v > 1$

13. Consider the region bounded by the graphs of  $y = (x \arctan x)^{1/2}$  and  $y = 0$ , for  $0 \leq x \leq 1$ . Find the volume of the solid formed by rotating this region about the **x-axis**.

*I'm very good at integral and differential calculus,  
I know the scientific names of beings animalculous;  
In short, in matters vegetable, animal, and mineral,  
I am the very model of a modern Major-General.  
About binomial theorems I'm teeming with a lot of news,  
With many cheerful facts about the square on the hypotenuse.*

- W. S. Gilbert, **The Pirates of Penzance**(1879)