MATH 162 PRACTICE TEST III

1. For each series below, determine absolute convergence, conditional convergence or divergence. Justify each answer.

$$(a)\quad \sum_{n=3}^{\infty} (-1)^n \frac{13}{(\ln n)^{13}}$$

$$(b)\quad \sum_{k=1}^{\infty} (-1)^k \frac{(k+3)(k^2+5)}{(k+13\ln k)^4}$$

$$(c)\quad \sum_{n=1}^{\infty} (-1)^n \frac{2^n}{\left(1+\frac{1}{n}\right)^{n^2}}$$

$$(d)\quad \sum_{n=1}^{\infty} (-1)^n \frac{1}{\ln(e^n+e^{-n})}$$

$$(e)\quad \sum_{n=1}^{\infty} (-1)^n \frac{n^{13}}{(n+13)!}$$

2. For each power series below, determine the *radius of convergence* and the *interval of convergence*. Study the behavior of each power series at the *endpoints*.

(a) $$\sum_{n=1}^{\infty} \frac{13^n}{n(n+13)} x^n$$

(b) $$\sum_{n=1}^{\infty} \frac{1}{n(n+3)(n+11)} (x-4)^n$$

(c) $$\sum_{n=1}^{\infty} \frac{1}{\sqrt{3n+7}} (x+13)^n$$

3. (a) Find the 3rd order Maclaurin polynomial of *cosh x*.

(b) Find the 5th order Taylor polynomial of *cos x* centered at $x = \pi/2$.

4. Find the 4th order Taylor polynomial of e^x centered at $x = 2$.

5. Find the 3rd order Maclaurin polynomial of

$$f(x) = 4 + (x+13)^2 + (x+13)^3$$

6. By differentiating the power series expansion of $y = 1/(1 - x)$, find the value of

$$\sum_{k=0}^{\infty} \frac{k}{13^k}$$

7. Find the *first five* non-zero terms of the Maclaurin series expansion of

$$h(x) = (1 + 2x^2)\, e^{3x}.$$

8. Let $f(x) = x^8 e^{5x}$. Compute $f^{(100)}(0)$. Do not simplify your answer.

9. Find the *radius of convergence* of the power series:

$$\sum_{n=1}^{\infty} \frac{n!}{(1)(3)(5)(7)...(2n-1)} x^n$$

10. Find the *radius of convergence* of the power series:

$$\sum_{n=0}^{\infty} n!\, x^{2^n}$$

11. Find the *radius of convergence* of the power series:

$$\sum_{n=1}^{\infty} \frac{1}{(\ln n)^n} x^n$$

12 . *Without using* l'Hôpital's rule, calculate the following limit. Show your work!

$$\lim_{t \to 0} \frac{te^{4t} - \sin(3t) + 2t - 4t^2}{t^3}$$

13. Let $G(x) = x^3 \cosh(3x)$. Using an appropriate Maclaurin series, compute $G^{(2015)}(0)$. (Do not try to simplify your answer.)

14. Find the first four non-zero terms of the Maclaurin series for each of the following:

(a) $\dfrac{e^{2x}}{\cosh x}$

(b) $\dfrac{\ln(1+x)}{1+x^2}$

(c) $e^{x^2} \sin 2x$

15. (a) Express $\left(\dfrac{13+i}{1+i}\right)(1-2i)+4+5i$ as a complex number of the form a + bi.

(b) By expressing -1 as an appropriate complex power of *e*, calculate the five fifth roots of -*i*.

Express your answers in the form a + bi.

(c) Using Euler's formula, express cos(4x) in terms of cos x and sin x.

16. Using division of power series, find the first three non-zero terms of the Maclaurin series expansion of

$$f(x) = \frac{e^{2x}+1}{\cos x}$$

17. Using multiplication of power series, find the first four non-zero terms of the Maclaurin series expansion of

$$g(x) = e^{x^2}(1+x^2+x^3)$$

18. Determine the *interval of convergence* of the following power series. (You need not study end-point behavior.)

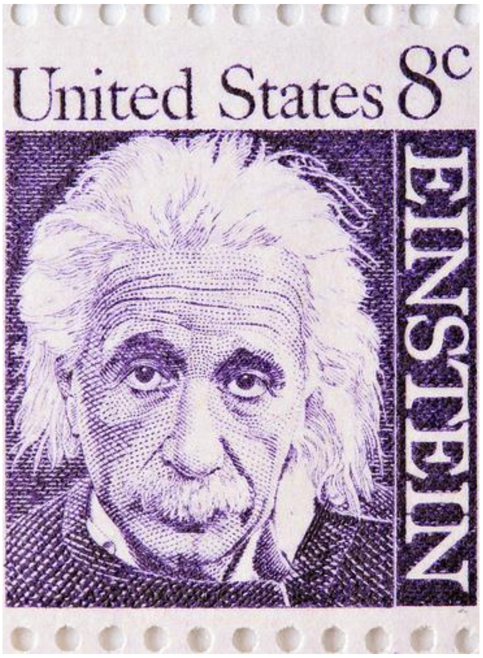
$$\sum_{n=1}^{\infty} \frac{n^{13}\, 13^n}{\sqrt{n+2015}} (x-13)^n$$

19. Analyze the behavior of the series

$$\sum_{n=1}^{\infty} \frac{\sqrt{n^2+4}}{\left(n^{1/3}+1789\right)^5}$$

20. Prove Euler's formula.

21. What is the relationship between cosh x and cos x? between sinh x and sin x?



As far as the laws of mathematics refer to reality, they are not certain; as far as they are certain, they do not refer to reality.

\- Albert Einstein