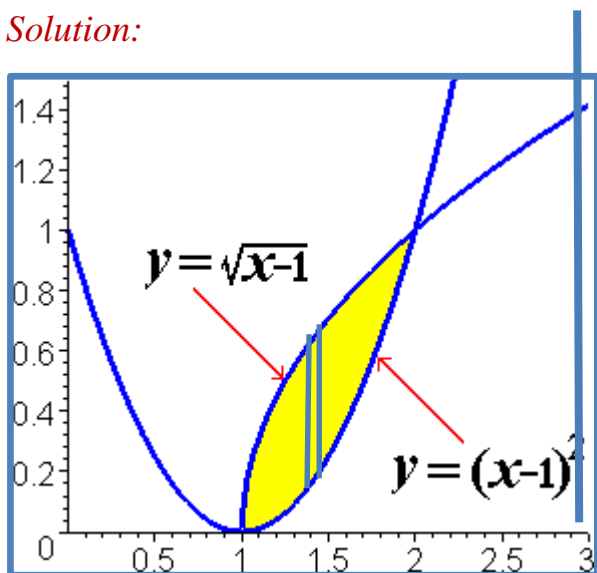


MATH 162

SOLUTIONS: QUIZ II

1. [20 pts] Consider the region \mathcal{R} in the first quadrant that is bounded by the curves $y = (x - 1)^2$ and $y = \sqrt{x - 1}$. Sketch! This region is rotated about the axis $x = 3$. Using the *shell method* write the volume of this solid of rotation as a Riemann integral.

Solution:



The thickness of the shell is Δx , so the integrand must be a function of x . The distance of the shell from the axis of rotation is $3 - x$. Furthermore, the height of the shell is the difference between the two y -values (corresponding to x) which is determined by the two curves.

That is, the height of the shell is

$$\sqrt{x - 1} - (x - 1)^2$$

Hence the volume of the solid of revolution can be expressed by the integral:

$$2\pi \int_1^2 (\sqrt{x-1} - (x-1)^2)(3-x) dx$$

2. [10 pts] Consider the parameterized curve: $x = e^t - 1$, $y = e^{2t}$, where $t \geq 0$, describing the position in the xy -plane of Mehitabel, the cat, at time t .

(a) By eliminating the parameter, t , express y as a function of x .

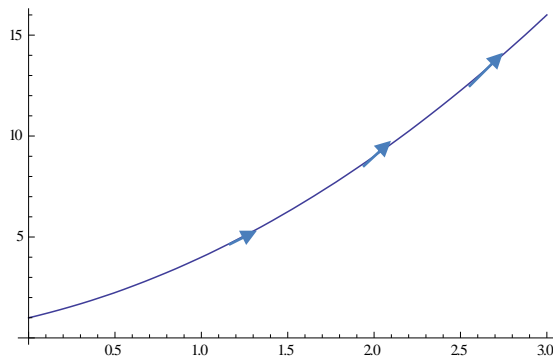
Solution: Since $x = e^t - 1$, $e^t = x + 1$. Hence $y = e^{2t} = (x + 1)^2$. Now, the domain of this function is $[0, \infty)$.

- (b) *Sketch* the parameterized curve, using an arrow to indicate the direction of Mehitabel's journey. Also, indicate the birthplace of Mehitabel.

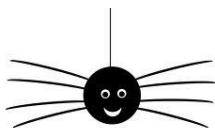


Solution: When $t = 0$, $x = 0$ and $y = 1$. Thus Mehitabel is born at the point $(0, 1)$ in the xy -plane.

As $t > 0$ increases, both x and y increase. Thus Mehitabel's direction of motion is northeast.



3. [5 pts] Charlotte travels along a straight line from $P = (1, -3)$ to $Q = (5, 7)$. Find a parameterization for this trip. Let us assume that, when $t = 0$, Charlotte is at P and, when $t = 1$, Charlotte has reached Q .



Solution: Let $(x(t), y(t)) = (1 - t)(1, -3) + t(5, 7)$

Thus $x(t) = (1 - t)1 + 5t = 4t + 1$. and $y(t) = (1 - t)(-3) + 7t = 10t - 3$

The book of nature is written in the language of mathematics.

- Galileo