MATH 162 SOLUTIONS: QUIZ III

1. *[10 pts]* Albertine, a rock climber, is about to haul up 80 newtons (about 18 lbs) of equipment that has been hanging beneath her on 50 meters of rope that weighs 0.8 newton per meter. How much *work* will it take? Express your answer to the nearest joule.

Solution: Assume that Albertine's (fixed) position on the y-axis is y = 50 and that the equipment initially is at the origin. Then the work required to haul up the equipment alone is (50)(80) = 4000 joules.

To haul up the rope, the work required is:

$$W = \int_0^{50} 0.8(50-y)\,dy = 0.8\left(50y - \frac{y^2}{2}\right)\Bigg|_0^{50} = 1000 \text{ joules}$$

Thus the total amount of work performed in this task is 5000 joules.

2. *[10 pts]* Compute the *arc length* of the parameterized curve

$$x(t) = e^t + e^{-t},\quad y(t) = 5 - 2t \quad \text{where } 0 \le t \le 1$$

Express your answer as a Riemann integral. *No need to evaluate the integral.*

Solution: Since $dx/dt = e^t - e^{-t}$ and $dy/dt = -2$,

$$s = \int_0^1 \sqrt{(dx/dt)^2 + (dy/dt)^2}\,dt = \int_0^1 \sqrt{(e^t - e^{-t})^2 + (-2)^2}\,dt =$$

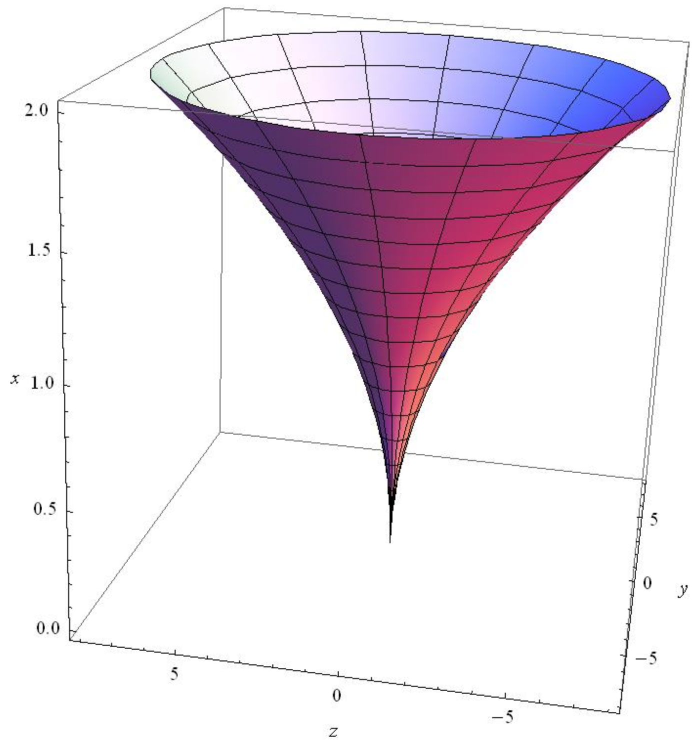
$$\int_0^1 \sqrt{e^{2t} - 2 + e^{-2t} + 4}\,dt = \int_0^1 \sqrt{e^{2t} + 2 + e^{-2t}}\,dt =$$

$$\int_0^1 \sqrt{(e^t + e^{-t})^2}\,dt = \int_0^1 (e^t + e^{-t})\,dt =$$

$$\left(e^t - e^{-t}\right)\Big|_0^1 = e - e^{-1} - (1-1) = e - e^{-1} \approx 2.35$$

3. *[10 pts]* A tank is designed by revolving the parabola $y = 2x^2$, $0 \le x \le 2$, about the y-axis. The tank, with dimensions in feet, is filled with fluid weighing 41 lbs/ft^3. How much *work* will it take to empty the tank by pumping water to the tank's top? Give your answer to the nearest ft-lb. *Sketch!*

Solution:



Let y denote any point in the interval [0, 8]. Consider a horizontal slice (passing through y) of our solid that has thickness Δy. *The area of the slice (circle with radius* $x = (y/2)^{1/2}$*) is* $\pi x^2 = \pi y/2$. *Hence the volume of the slice is* $(\pi y/2)\Delta y$ *(cubic feet). The weight of this slice is* $(\pi y/2)(\Delta y)(41)$ *lbs.*

This slice is $(8 - y)$ *feet from the top of the tank. Thus the work done in moving this slice to the top of the tank is:*

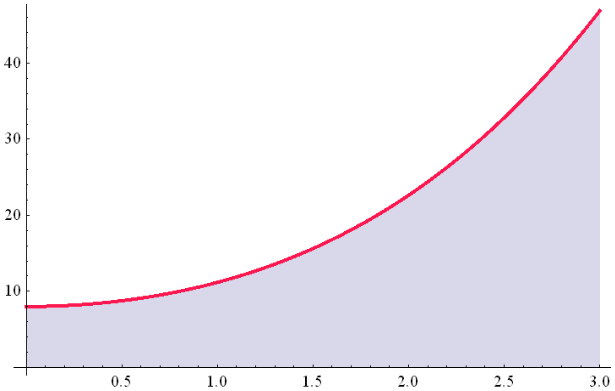
$$\Delta W = (8 - y)(\pi y/2)(41)\Delta y$$

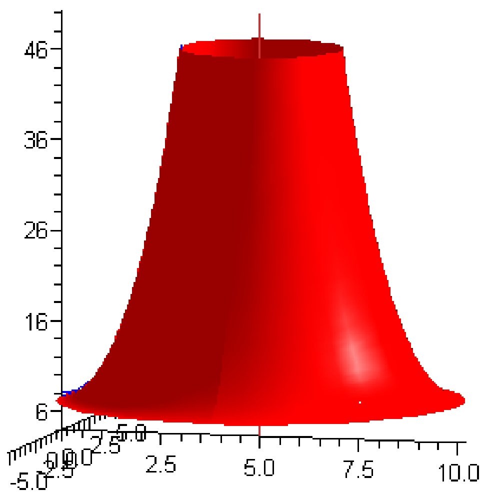
Summing and taking the limit as $\Delta y \to 0$, *we obtain:*

$$W = \int_0^8 (8 - y)\frac{\pi y}{2}(41)\,dy = \frac{41}{2}\int_0^8 (8 - y)\,y\,dy$$

4. *[10 pts]* The curve given by $y=\left(x^{2}+4\right)^{\frac{3}{2}}$ *for* $0\le x\le 3$ is rotated about the axis x = 5. Express the *area* of the generated surface as a Riemann integral. *Sketch! Do not evaluate the integral.*

Solution:





Note that ρ, the radius of revolution, is $5-x$. Thus

$$S=\int_{x=0}^{x=3}2\pi\rho\,ds=2\pi\int_{x=0}^{x=3}(5-x)\sqrt{1+\left(\frac{dy}{dx}\right)^{2}}\,dx=2\pi\int_{0}^{3}(5-x)\sqrt{1+\left(\frac{3}{2}(x^{2}+4)^{\frac{1}{2}})2x)\right)^{2}}\,dx=$$

$$2\pi\int_{0}^{3}(5-x)\sqrt{1+9x^{2}(x^{2}+4)}\,dx=2\pi\int_{0}^{3}(5-x)\sqrt{9x^{4}+36x^{2}+1}\,dx$$

The book of nature is written in the language of mathematics.
- Galileo