# Math 162 Solutions: QUIZ V

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|  | ***Part I:*** Select *any three* of the following four integrals. For each improper integral that you select, determine convergence or divergence. *Justify your answers!* (You may answer all four for extra credit.)    *Solution: This integral diverges since:*      *Solution: Note that the dominant term in the numerator is 1 (not x15).*  *This integral converges because:*    *Using the p-test, we know that  converges. Hence, by the comparison test,*  *converges*.    *Solution: This integral diverges because:*    *since cos(2c)→ 0+ as c → (/4)-.*    *Solution: This integral of mixed type converges because, by definition:*    *and each of these two integrals converges (by virtue of the Comparison Test):*  *For 0 < x ≤ 1:*    *and by the p-test for integrals of type II,*    *For x ≥ 1:*    *and by the p-test for integrals of type I,*    ***Part II:*** For each of the following *sequences*, determine *convergence* or *divergence*. In the case of convergence, find the *limit* of the sequence. Briefly justify each answer. *(Select any 7 of the 8 sequences. For extra credit, you may solve all eight.)*    *Solution: Since n = o(n!) and ln n = o(n), {an} converges and its limit is 0.*    *Solution: Using the fact that the limit of the sum of two convergent sequences is the sum of their limits, we have:*    *Thus the sequence {bn} converges and its limit is e4.*    *Solution: Since -1 ≤ sin(4n) ≤ 1, we have:*    *Applying the Squeeze Theorem, we conclude that {cn} converges to 0.*    *Solution: Observing that dn ≥ nn/ (n14 + n14) = ½ nn/ n14 = ½ nn – 14 → ∞ as*  *n → ∞, we conclude that dn is unbounded, and thus divergent.*  (e) en = (-1)n cos(1/n)  *Solution: First note that, as n → ∞, cos(1/n) → cos 0 = 1.*  *Thus for large n, en is approximately (-1)n which is a divergent sequence.*    *Solution: By selecting the dominant terms, we have:*    *Hence we conclude that {An} converges to 12/5.*  (g) *Bn* = arctan (ln(n))  *Solution: As n → ∞, ln n → ∞. and hence:*    *Thus the sequence {Bn } converges and its limit is /2.*    *Solution: Rationalizing this expression:*    *Thus the sequence {Cn } converges and its limit is 3.* |

*Mathematicians have tried in vain to this day to discover some order in the sequence of prime numbers, and we have reason to believe that it is a mystery into which the human mind will never penetrate.*

- Leohard Euler (1707-1783)