# Math 162 Solutions: QUIZ V

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|  | ***Part I:*** Select *any three* of the following four integrals. For each improper integral that you select, determine convergence or divergence. *Justify your answers!* (You may answer all four for extra credit.) *Solution: This integral diverges since:**Solution: Note that the dominant term in the numerator is 1 (not x15).* *This integral converges because:**Using the p-test, we know that  converges. Hence, by the comparison test,*  *converges*.*Solution: This integral diverges because:* *since cos(2c)→ 0+ as c → (/4)-.**Solution: This integral of mixed type converges because, by definition:**and each of these two integrals converges (by virtue of the Comparison Test):**For 0 < x ≤ 1:**and by the p-test for integrals of type II,**For x ≥ 1:**and by the p-test for integrals of type I,****Part II:*** For each of the following *sequences*, determine *convergence* or *divergence*. In the case of convergence, find the *limit* of the sequence. Briefly justify each answer. *(Select any 7 of the 8 sequences. For extra credit, you may solve all eight.)**Solution: Since n = o(n!) and ln n = o(n), {an} converges and its limit is 0.* *Solution: Using the fact that the limit of the sum of two convergent sequences is the sum of their limits, we have:**Thus the sequence {bn} converges and its limit is e4.**Solution: Since -1 ≤ sin(4n) ≤ 1, we have:**Applying the Squeeze Theorem, we conclude that {cn} converges to 0.**Solution: Observing that dn ≥ nn/ (n14 + n14) = ½ nn/ n14 = ½ nn – 14 → ∞ as* *n → ∞, we conclude that dn is unbounded, and thus divergent.*  (e) en = (-1)n cos(1/n)*Solution: First note that, as n → ∞, cos(1/n) → cos 0 = 1.* *Thus for large n, en is approximately (-1)n which is a divergent sequence.**Solution: By selecting the dominant terms, we have:**Hence we conclude that {An} converges to 12/5.*(g) *Bn* = arctan (ln(n))*Solution: As n → ∞, ln n → ∞. and hence:**Thus the sequence {Bn } converges and its limit is /2.**Solution: Rationalizing this expression:**Thus the sequence {Cn } converges and its limit is 3.* |

*Mathematicians have tried in vain to this day to discover some order in the sequence of prime numbers, and we have reason to believe that it is a mystery into which the human mind will never penetrate.*

 - Leohard Euler (1707-1783)