MATH 162 SOLUTIONS: QUIZ IX

1. *[50 pts]* Answer any five of the following problems. You may answer all six to earn extra credit. Find the *indefinite integral* of each of the following functions:

(*a*) $\tan^5 x$

Solution:

$$\int \tan^5 x\, dx = \int (\tan^3 x)(\sec^2 x - 1)dx = \int (\tan^3 x)(\sec^2 x)dx - \int (\tan^3 x)dx = \frac{\tan^4 x}{4} - \int \tan^3 x\, dx =$$

$$\frac{\tan^4 x}{4} - \int \tan^3 x\, dx = \frac{\tan^4 x}{4} - \int (\tan x)(\sec^2 x - 1)dx = \frac{\tan^4 x}{4} - \int (\tan x)(\sec^2 x)dx + \int \tan x\, dx =$$

$$\frac{\tan^4 x}{4} - \frac{\tan^3 x}{3} - \ln|\cos x| + C$$

(*b*) $\sec^{2015} x \tan x$

Solution:

$$\int \sec^{2015} x \tan x\, dx = \int (\sec^{2014} x)(\sec x \tan x)\, dx = \frac{\sec^{2015} x}{2015} + C$$

(*c*) $\sin^2(3x)\ \cos^3(3x)$

Solution:

$$\int \sin^2(3x)\ \cos^3(3x)\ dx = \int \sin^2(3x) \cos^2(3x)\ \cos(3x)\ dx =$$

$$\int \sin^2(3x) \cos^2(3x)\ \cos(3x)\ dx = \int \sin^2(3x)\left(1 - \sin^2(3x)\right) \cos(3x)\ dx$$

Next, let t = cos 3x.

(*d*) $x^3\sqrt{x^2+1}$

Solution: Let $t = x^2$. Then dt = 2x dx. So x dx = (1/2) dt

$$\int x^3\sqrt{x^2+1}\ dx = \int x^2\sqrt{x^2+1}\ (x\,dx) =$$
$$\int t\sqrt{t+1}\ (1/2)\,dt = (1/2)\int t\sqrt{t+1}\ \ dt$$

Next, let y = t + 1.

(e) $\dfrac{e^{2x}}{1+e^{4x}}$

Solution: Let u = e^{2x}. Then du = $2e^{2x}$ d.x So e^{2x} dx = ½ du

$$\int \frac{e^{2x}}{1+e^{4x}}\ dx = \int \frac{(1/2)\,du}{1+u^2}\,du = (1/2)\arctan u + C =$$

$$(1/2)\arctan\left(e^{2x}\right) + C$$

(f) $\dfrac{1}{(\arctan x)\ (1+x^2)}$

Solution: Let u = arctan x. Then du = 1/(1+x^2) du. Hence:

$$\int \frac{1}{(\arctan x)\ (1+x^2)}\ dx = \int \frac{1}{u}\,du = \ln u + C = \ln(\arctan x) + C$$

2. *[12 pts] Express in the form a + bi. Simplify fully*

(i) $(\mathrm{i}\ )^{-\mathrm{i}}$

Solution:

$$i^{-i} = \left(e^{(\pi/2)i}\right)^{-i} = e^{(\pi/2)(-i^2)} = e^{\pi/2}$$

(ii) $e^{(7\pi/6)i}$

Solution:

$$e^{(7\pi/6)i} = \cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6} = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$$

(iii) $81\, e^{(2015\pi)i}$

Solution:

$$e^{2015\pi i} = e^{(2014+1)\pi i} = e^{\pi i} = -1$$

3. *[10 pts]* *Using Euler's formula*, find a formula for sin (4x) in terms of sin x and cos x.

Solution:

Since $e^{xi} = \cos x + i \sin x$, *we have:*

$e^{4xi} = (\cos x + i \sin x)^4 = (\cos x)^4 + 4(\cos x)^3\, i \sin x + 6(\cos x)^2\, i^2 (\sin x)^2 +$
$4(\cos x)\, i^3 (\sin x)^3 + (i^4)(\sin x)^4 =$
$(\cos x)^4 + 4(\cos x)^3 (\sin x)\, i - 6(\cos x)^2(\sin x)^2 - 4(\cos x)(\sin x)^3\, i + (\sin x)^4$

But Euler's formula tells us that $e^{4xi} = \cos(4x) + i \sin(4x)$.

Thus $\sin(4x) = 4(\cos^3 x)(\sin x) - 4(\cos x)(\sin x)^3$

The imaginary number is a fine and wonderful resource of the human spirit, almost an amphibian between being and not being.

- Gottfried Wilhelm Leibniz (1646-1716)