

FINAL EXAM REVIEW: MATH 162

Thomas's Calculus: Chapters 5 (sections 5 & 6), 6 (sections 1 – 5), 7, 8, 10, 11 (sections 1 & 2), Appendix A.7

1. Review of Calculus I

(A) Understanding graphs

- (1) graphing rational functions
- (2) regions of increase and decrease
- (3) concavity
- (4) limiting behavior, singularities
- (5) first and second derivative tests

(B) Limits

- (1) $(\sin x)/x \rightarrow 0$ as $x \rightarrow 0$
 - (2) l'Hôpital's rule
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(C) Definition and meaning of the definite (Riemann) integral.

- (1) area between curves
- (2) distance, velocity, acceleration problems
- (3) limit of a Riemann sum equals a definite integral
- (4) average value of a function
- (5) properties of the indefinite and the definite integral

(D) Fundamental Theorem of Calculus

- (1) the two versions of the theorem
- (2) using the chain rule and the FTC to differentiate integrals

2. Volumes of solids

- (A) Cavalieri's Principle: Solids with known cross-section (square, triangle, semi-circle, etc.)
- (B) solids of revolution
 - (1) disk and washer method
 - (2) shell method
- (C) arc length
- (D) surface area
- (E) work

3. Transcendental functions

- (A) differentiating and integrating functions of the form b^x and $f(x)^{g(x)}$
- (B) relative growth rates: the race to infinity; little o and big O notation
- (C) **hyperbolic functions**
 - (1) graphs
 - (2) identities
 - (3) inverse

4. Techniques of integration

- (A) judicious guessing
- (B) substitution (for both indefinite and definite integrals)
- (C) use of symmetry in evaluating definite integrals (odd functions, even functions)
- (D) integration by parts
- (E) trigonometric integrals
- (F) trigonometric substitutions
- (G) partial fraction decomposition
- (H) verifying an integration formula by means of differentiation
- (I) recursive (or reduction) integration formulas

Theory

- (1) limit of $(1 + c/x)^x$ as x increases without bound equals e^c .

5. Improper Integrals

- (A) first kind, second kind
- (B) using the Comparison Test
- (C) Gamma function and Stirling's formula
- (D) probability density functions and random variables

6. Numerical sequences and series

- (A) convergence, divergence of sequences
- (B) finding the limit of a convergent sequence
- (C) recursively defined sequences
 - (1) Fibonacci sequence
 - (2) sequences of the form $a_n = (a_{n-1} + k/a_{n-1})/2$
- (D) telescoping series
- (E) *geometric series*
- (F) harmonic series
- (G) n^{th} -term test for Divergence of a series
- (H) integral Test
- (I) comparison test for series
- (J) limit comparison test
- (K) ratio test
- (L) root test
- (M) absolute and conditional convergence
- (N) alternating series and the Cauchy-Leibniz rule

Theory

- (1) statement of the Comparison Test for improper integrals
- (2) properties of convergent sequences
- (3) statement of the squeeze theorem
- (4) properties of convergent series
- (5) proof of geometric series result
- (6) proof of divergence of the harmonic series

(7) alternating series estimation theorem: If $\sum (-1)^n a_n$ satisfies the hypotheses of Leibniz's rule and converges to S , then the error between the n^{th} partial sum of the series and S is less than or equal to a_{n+1} .

(8) $b^n / n! \rightarrow 0$ as $n \rightarrow \infty$

7. Power series

- (A) radius of convergence
- (B) interval of convergence
- (C) power series representation of a function
- (D) differentiation and integration of power series
- (E) multiplication and division of power series

8. Taylor and Maclaurin series

- (A) computing n^{th} order Taylor and Maclaurin polynomials of a function
- (B) computing Taylor and Maclaurin series of a function
- (C) recognizing the series of *familiar* functions: $1/(1-x)$, $\sin x$, $\cos x$, e^x , $\sinh x$, $\cosh x$, $\ln(1+x)$
- (D) short-cuts for computing series: e.g., Maclaurin series of $\sin x - 9e^{3x}$, $x^3/(1+x^4)$, $\sin(x^5)$, $x^4 \cosh(3x)$, $\arctan x$
- (E) using Taylor series to compute limits (instead of L'Hopital's rule)
- (F) using Taylor series of $g(x)$ at $x = c$ to compute higher derivatives of $g(c)$.
- (G) Taylor's formula with remainder; estimating the remainder
- (H) binomial series

Theory

- (1) find $\sum (n/b^n)$ or $\sum (n^2/b^n)$ using differentiation of an appropriate power series.
- (2) binomial Theorem, Pascal's triangle

9. Complex numbers

- (A) Euler's formula
- (B) de Moivre's theorem

- (C) relationships among $\cosh x$, $\sinh x$ and $\cos x$, $\sin x$
- (D) finding the n^{th} roots of unity
- (E) deriving trig identities, such as expressing $\cos 5x$ in terms of $\cos x$ and $\sin x$

The other one, the one called Borges, is the one things happen to. I walk through the streets of Buenos Aires and stop for a moment, perhaps mechanically now, to look at the arch of an entrance hall and the grillwork on the gate; I know of Borges from the mail and see his name on a list of professors or in a biographical dictionary. I like hourglasses, maps, eighteenth-century typography, the taste of coffee and the prose of Stevenson; he shares these preferences, but in a vain way that turns them into the attributes of an actor. It would be an exaggeration to say that ours is a hostile relationship; I live, let myself go on living, so that Borges may contrive his literature, and this literature justifies me. It is no effort for me to confess that he has achieved some valid pages, but those pages cannot save me, perhaps because what is good belongs to no one, not even to him, but rather to the language and to tradition. Besides, I am destined to perish, definitively, and only some instant of myself can survive in him. Little by little, I am giving over everything to him, though I am quite aware of his perverse custom of falsifying and magnifying things.

Spinoza knew that all things long to persist in their being; the stone eternally wants to be a stone and the tiger a tiger. I shall remain in Borges, not in myself (if it is true that I am someone), but I recognize myself less in his books than in many others or in the laborious strumming of a guitar. Years ago I tried to free myself from him and went from the mythologies of the suburbs to the games with time and infinity, but those games belong to Borges now and I shall have to imagine other things. Thus my life is a flight and I lose everything and everything belongs to oblivion, or to him. I do not know which of us has written this page.

- Jorge Luis Borges (1899-1986), excerpt from "Borges and I",
Labyrinths: Selected Stories and Other Writings (1964)



Henri Rousseau. **Exotic Landscape.** (1910)