# MATH 162 Solutions: TEST IIi

|  |  |  |
| --- | --- | --- |
|  |  |  |

"Then you should say what you mean," the March Hare went on.

"I do, " Alice hastily replied; "at least I mean what I say, that's the

same thing, you know."

"Not the same thing a bit!" said the Hatter. "Why, you might just as

well say that "I see what I eat" is the same thing as "I eat what I see!"

* Lewis Carroll, **Alice in Wonderland**

**Instructions:** *Answer any 7 of the following 8 problems. You may answer all 8 to obtain extra credit.*

1. Without using l'Hôpital’s rule, find:



*Solution:*

*Since*



*it follows that:*



*Since*



*it follows that:*



*Hence:*



2. Given y = G(x) below, calculate the value of G(1313)(0). *(Express your answer in factorial form.)*



*Solution:*

*Beginning with the Maclaurin series for sinh t and then replacing t by x2:*



*Now, multiplying by x3 yields:*



*Now, the general Maclaurin series of G(x) is:*

**

*Thus the coefficient of x1313 is G(1313)(0) / 1313!*

*Now the series for x3 sinh (x2) has coefficient of x1313 occur when 4n + 5=1313, that is, when n =327 (and so 2n + 1=655). Thus this coefficient is: 1 / 655!*

*Equating G(1313)(0) / 1313! with 1 /655!, we find that:*

***G(1313)(0) = 1313! / 655!***

**3**. By dividing power series, find the *first three non-zero* terms of the Maclaurin series of



*Solution:*



**4.**For each series below, determine *absolute convergence*, *conditional convergence* or *divergence*. Justify each answer.



*Solution:*

*Notice that this series fails to converge absolutely, by the p-test. It does converge, however, due to the Cauchy-Leibniz test. Thus the series converges conditionally.*



*Solution:*

*Since arctan(k2) → /2 as k → ∞, the series diverges by the nth-term test for divergence.*



*Solution:*

*Applying the Ratio Test, we see that the series converges absolutely:*



5. Write each of the following in the form a + bi. *Show your work!*

1. 3(9 – 4i) – 5(-6 – 3i)

*Answer: 57 + 3i*

1. (1 – i)(2 – 5i)

*Answer: -4 – 7i*

1. (3 – i)3

*Answer: 18 – 26 i*



*Answer: 14 + 345i*



*Answer:* 

(g) i1789 + i444 – i9902

*Answer:* i1789 + i444 – i9902 = i4(447)+1 + i4(111) – i4(247) +2 = i + 1 + 1 = 2 + i

**6.** For each power series below, determine the *interval of convergence*. *Do not* investigate the behavior of at endpoints.

 

*Solution:*

*Using the ratio test:*



*Thus the series converges absolutely for |x – 13| < 13. So the interval of convergence is* 

(b) 

*Solution:*

*Invoking the nth root test:*



*Thus the series converges absolutely for e |x – 4| < 1. So the interval of convergence is (4 – 1/e, 4 + 1/e).*

**7.** For each power series below, determine the *interval of convergence*. Investigate *end point behavior*.



*Solution:*

*Using the ratio test:*



*Thus the series converges absolutely for |x – 13| < 1. So the interval of convergence is (12, 14).*

*At x = 14, the series equals:*

**

*which diverges (using the comparison test and the p-test).*

*At x = 12, the series equals:*

**

*which converges conditionally (using the Cauchy-Leibniz test as well as the fact that the series fails to converge absolutely).*

 

*Solution:*

*Using the ratio test:*



*Thus the series converges absolutely for 13x2 < 1. So the interval of convergence is* 





*which converges absolutely (using the p-test).*





*which converges absolutely (using the p-test).*

8. Find the *interval of convergence* of each of the following power series:



*Solution: Using the ratio test:*



*Now, the series converges absolutely when ½ |x|3 < 1.*

*Thus the interval of convergence of our series is:*





*Solution: Applying the ratio test,*



*Thus the interval of convergence of our series is*



***Extra Credit:*** Using a series representation of sin(3x), find constants

*r* and *s* for which:



*Solution:*

*Since*



*we have:*



*If this limit equals 0, then 3 + r = 0 and* s – 33/(3!) = 0.

*Hence r = -3 and s = 9/2*