**WORKSHEET X**

sequences

1.   Explain precisely what it means for a sequence {an} to *converge*.  What does it mean to say that a sequence *diverges*?  What is meant by the *limit* of a sequence?

2.    Discuss the rules for convergence (and divergence) of a sum, difference, product, and quotient of two sequences. State the Squeeze (or Sandwich) Theorem for sequences. What is a *monotonic sequence*? What can be said about an increasing sequence that is bounded above? Is every bounded sequence convergent? Is every convergent sequence bounded?

3. Explain why the limit of a convergent sequence must be unique.

4.   For each of the following sequences, {tn}, determine convergence or divergence.  If the sequence converges, are you able to find its limit?

1. an = (-1)n
2. tn =  sin (n/2)
3. tn =  cos(n)
4. tn =  (ln n) / n
5. tn =  3 arc tan (n2)
6. tn =  (ln n) / (ln ln n)
7. tn =  (1 + 1/n)n
8. tn = (-1)n+1 n2
9. tn = ½ + (-1)n/2
10. tn = n! / (n+3)
11. tn =   (cosh n) / (sinh n)
12. tn =  1/2 + 1/3 + 1/4 + ... + 1/n
13. tn =  (n4 – 3n2 + n5 + 13) / (n3ln n – 5n5 + ln(1+n6) – 99)
14. tn =  (sin n) / n
15. tn = 1/n!
16. tn = 2n / n!
17. tn = n! / 2n
18. tn = (n2 + 1)1/2 – n
19. tn = (n2 + n + 1)1/2 – n
20. tn = (n2 + 5n + 1)1/2 – (n2 + n + 1)1/2
21. tn = n sin(1/n)
22. tn = ln(n+1) – ln n
23. tn = ln(n3+n+1) – ln(n2–n+5)
24. tn =  n1/n
25. tn = (1 + 3/n)n
26. bn = n! / nn

5.    For each of the following *recursively defined sequences*, do you believe that it converges or diverges? Give evidence.  In the former case, assume that the limit exists and find it.

(a)    t1 = 1, t2 = 1,  tn = tn-1 + tn-2 for all n ≥ 3 (Of what value is the Fibonacci sequence?)  
(b)     a1 = 4,   an = an-1 /2 for all n ≥ 2   
(c)      c1 = 3,  cn = 1.01(cn-1) for all n ≥ 2   
(d)     b1 = 1,  bn =  (bn-1 + 3/bn-1)/2   for all n ≥ 2   
(e)      h1 = 1,  hn = hn-1 + 1/n  for all n ≥ 2   
(f)       a1 = 1, a2 = 2, an = (an-1 + an-2)/2 for all n ≥ 3.   
(g)      z1 = 1/3,  zn = (zn-1)2 for all n ≥ 2.

6. Suppose that a sequence {xn} is defined recursively by:

x0 = 1, x1 = 2, and xn+1 = 3xn + xn-1

Assuming that the limit of xn+1/xn exists, find it.

*Mathematicians have tried in vain to this day to discover some order in the sequence of prime numbers, and we have reason to believe that it is a mystery into which the human mind will never penetrate.*

* Leonhard Euler (1707-1783)

*At 6 P.M. the well marked 1/2 inch of water, at nightfall 3/4 and at daybreak 7/8 of an inch.  By noon of the next day there was 15/16 and on the next night 31/32 of an inch of water in the hold.  The situation was desperate.  At this rate of increase few, if any, could tell where it would rise to in a few days.*

* Stephen Leacock

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