WORKSHEET XIV: A BRIEF INTRODUCTION TO PROBABILITY

Definitions:

- f(x) is a *probability density function* (pdf) for X if

  the probability that $a \le X \le b$ is $\int_a^b f(x)\,dx$

- The *Mean* value for X with probability density function f(x) is

  $$\mu = \int_{-\infty}^{\infty} x\, f(x)\,dx \quad \text{(the weighted average value of x)}$$

- The *Median* value for X with probability density function f(x) is a value T such that

  $$\int_{-\infty}^{T} f(x)\,dx = \int_{T}^{\infty} f(x)\,dx = \frac{1}{2}$$

The probability density function for the *exponential distribution* in general (where $\lambda > 0$):

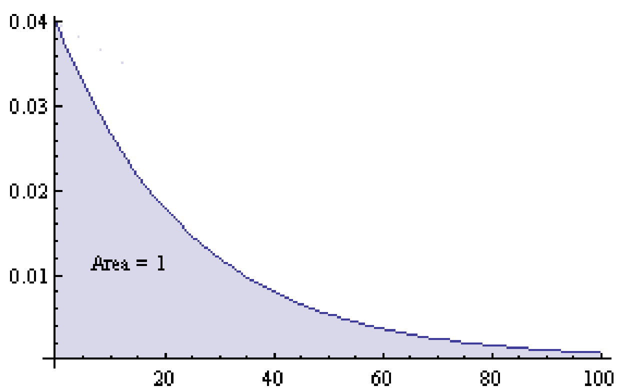
$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x > 0 \\ 0 & \text{if } x \le 0 \end{cases}$$

The mean = $1/\lambda$

Example: The probability density function for the time until failure for a smart phone chip is given by

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ 0.04e^{-0.04x} & \text{if } x \ge 0 \end{cases}, \quad \text{where } x \text{ is measured in weeks.}$$

A. Sketch a graph of f(x), the pdf:



B. Find the probability that a chip fails in the first 5 weeks:

$$\text{Prob}(X \le 5) = \int_0^5 0.04e^{-0.04t}\,dt = 0.181$$

C. Find the probability that a chip does NOT fail in the first 5 weeks:

$$\text{Prob}(X \ge 5) = \int_5^{\infty} 0.04e^{-0.04t}\,dt = 1 - \int_0^5 0.04e^{-0.04t}\,dt = 1 - 0.181 = 0.819$$

D. Find the probability that the chip fails at a time between 5 and 20 weeks.

$$\text{Prob}(5 \le X \le 20) = \int_5^{20} 0.04e^{-0.04t}\,dt = 0.369$$

E. Find the probability that the chip lasts longer than 36 weeks:

$$\text{Prob}(X \ge 36) = \int_{36}^{\infty} 0.04e^{-0.04t}\,dt = 1 - \int_0^{36} 0.04e^{-0.04t}\,dt = 1 - 0.763 = 0.237$$

F. Find the *median failure time* for this chip:

median = T, where Prob(X ≤ T) = 0.5.

$$\text{Prob}(X \le T) = \int_0^{T} 0.04e^{-0.04x}\,dx = 0.5$$

Solve the equation for T :

$$-e^{-0.04x}\Big|_0^T = -e^{-0.04T} - (-e^0) = 1 - e^{-0.04T} = 0.5$$

$$1 - 0.5 = e^{-0.4T},\ 0.5 = e^{-0.4T},\quad \ln(0.5) = -0.04T$$

$$T = \frac{\ln(0.5)}{-0.04} = \frac{-\ln(2)}{-0.04} = \frac{\ln(2)}{0.04} = 17.32\ weeks$$

G. Find the *mean failure time* for this chip:

Mean = weighted average =

$$\int_{-\infty}^{\infty} xf(x)dx = \int_0^{\infty} x\left(0.04e^{-0.04x}\right)dx =$$

$$\lim_{c\to\infty}\left(-25e^{-0.04x} - xe^{-0.04x}\Big|_0^c\right)$$

$$= \lim_{c\to\infty}\left(-25e^{-c} - ce^{-c}\right) - \left(-25e^0 - 0\right)$$

$$= (0-0) - (-25-0) = 25\ weeks$$

The most important questions in life are, for the most part, really only problems of probability.

- Pierre Simon de La Place