

# WORKSHEET XVI

## OPERATIONS ON POWER SERIES

- Consider the power series  $f(x) = 1/(1-x) = 1 + x + x^2 + x^3 + \dots$ . What is its interval of convergence?
  - Find the series for  $f'(x)$  and  $f''(x)$ .
- Using the power series obtained in 1(b) for  $f'(x)$ , determine the sum of the series  $\sum_{n=1}^{\infty} \frac{n}{3^n}$
- The series  
$$\sin x = x - x^3/3! + x^5/5! - x^7/7! + x^9/9! - x^{11}/11! + \dots$$
converges to  $\sin x$  for all real  $x$ .
  - Find the first six terms of a series for  $\cos x$ . For which values should the series converge?
  - By replacing  $x$  by  $2x$  in the series for  $\sin x$ , find a series that converges to  $\sin 2x$  for all  $x$ .
  - Using series multiplication, find a series that converges to  $2 \sin x \cos x$ .
- On which interval does the series  $1/(1+t) = 1 - t + t^2 - t^3 + t^4 - t^5 + \dots$  converge?
  - Integrating both sides over the interval  $[0, x]$ , find the first six terms of a series that converges to  $\ln(1+x)$ .
- The series  $e^x = 1 + x + x^2/2! + x^3/3! + x^4/4! + x^5/5! + \dots$  converges to  $e^x$  for all real  $x$ .
  - Find a series for  $(d/dx) e^x$ .
  - Find a series for  $\int e^x dx$
  - In the series for  $e^x$ , replace  $x$  by  $-x$  to find a series expansion of  $e^{-x}$ .
  - In (c), replace  $x$  by  $x^2$  to find a series expansion of  $e^{-x^2}$ .

6. (a) Beginning with the series for  $1/(1 + x^2)$ , find a series expansion of  $\arctan x$ .

(b) Find a series expansion for  $\int \frac{\arctan x}{x} dx$ .

7. Using a series representation for  $\sin 3x$ , find values of  $r$  and  $s$  for which

$$\lim_{x \rightarrow 0} \left( \frac{\sin 3x}{x^3} + \frac{r}{x^2} + s \right) = 0$$

*I used to love mathematics for its own sake, and I still do, because it allows for no hypocrisy and no vagueness....*



- Stendhal, **The Life of Henri Brulard**