WORKSHEET XVIII

COMPLEX NUMBERS; EULER'S FORMULA

1. Let z = 2 + 3i and w = 6 – i. Compute and express each of the following in the form a + bi. Plot each number in the complex plane.

   (a) z + w

   (b) 5z

   (c) z – 3w

   (d) zw

   (e) 1/z

   (f) z/w

   (g) $\bar{z}$

   (h) $2z^2 + 1/w$

2. Find the *modulus* and *argument* of each of the following:

   (a) 1 + i

(b) -1 – i

(c) 1 + 2i

(d) 3 + 5i

3. Justify Euler's formula using power series.

4. Explain how de Moivre's theorem is a special case of Euler's formula.

5. Express z = 1 + i in polar form, $re^{i\theta}$. What is the *modulus* of z? What is its *argument*?

6. Express each of the following in the form a + bi

   (a) $e^{\pi i}$

   (b) i^i

   (c) $(1 + i)^{100}$

7. Using de Moivre's theorem, express sin 3θ in terms of sin θ and cos θ.

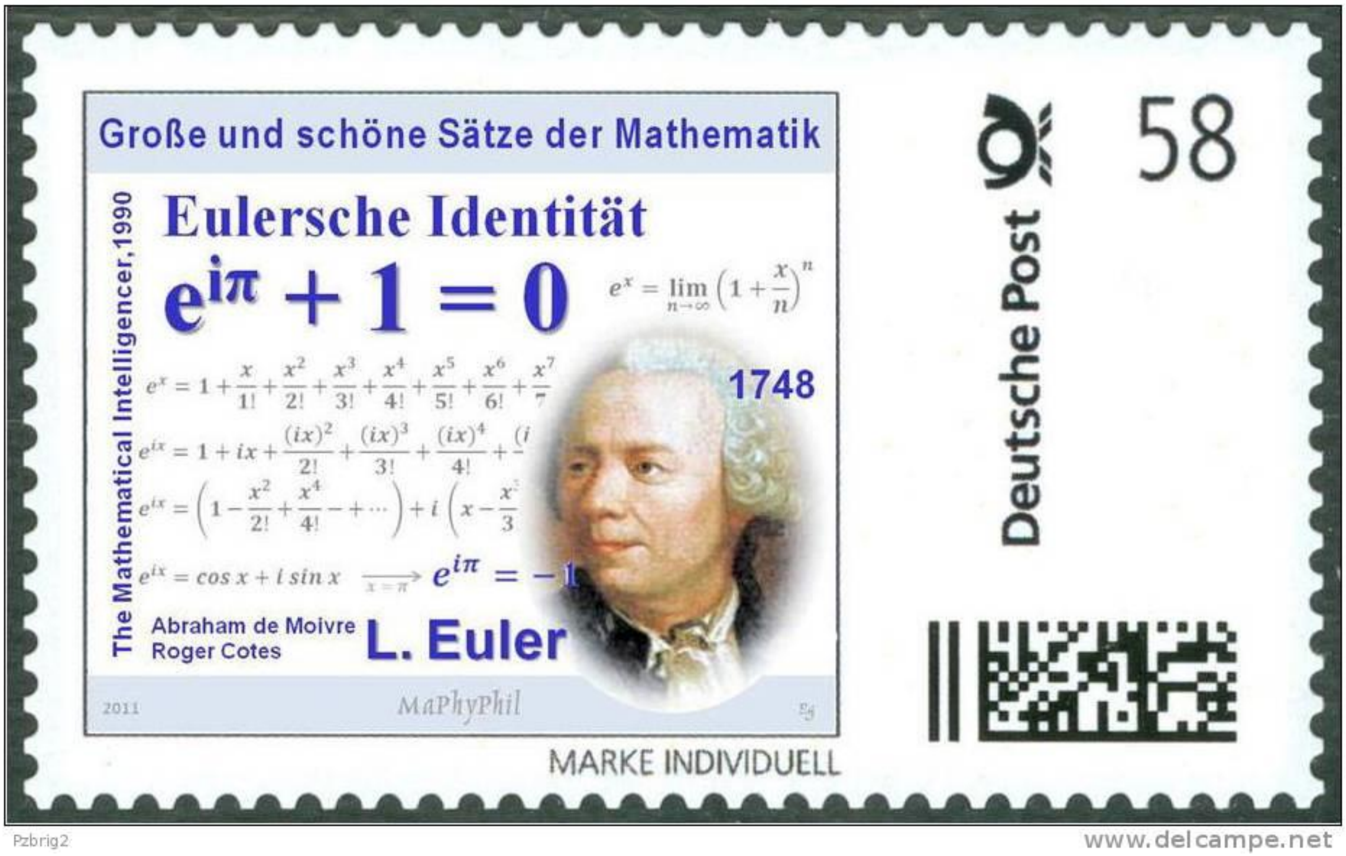
8. Using de Moivre's theorem, express cos 5θ in terms of sin θ and cos θ.

9. Find the four fourth roots of -1.

10. Find the three cube roots of 8i.

11. Find the five fifth roots of 1.

12. Using power series, determine a relationship between cosh x and cos x and between sinh x and sin x.



The imaginary number is a fine and wonderful resource of the human spirit, almost an amphibian between being and not being.

- Gottfried Wilhelm Leibniz (1646-1716)