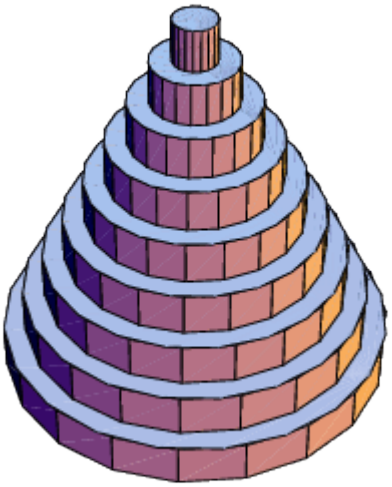
WORKSHEET II

VOLUME & CAVALIERI'S PRINCIPLE

1. The base of a solid is the region bounded by the curve $y = (\sin x)^{1/2}$ and the interval $[0, \pi]$ on the x-axis. The cross sections perpendicular to the x-axis are equilateral triangles with bases running from the x-axis to the curve. Find the volume of this solid.

2. The cross sections of a solid are squares perpendicular to the x-axis with their centers on the axis. If the square cut off at x has edge length of $2x^2$, find the volume of the solid between $x = 0$ and $x = a$.

3. Find the volume of a right circular cone of height *h* and base radius *r*. (*Hint:* Revolve an appropriate triangle about the x or y-axis.)



4. Consider the triangle T with vertices (0, 0), (2, 0), and (1, 1). Find the volume of the solid of revolution obtained by rotating T about:

(a) the x-axis

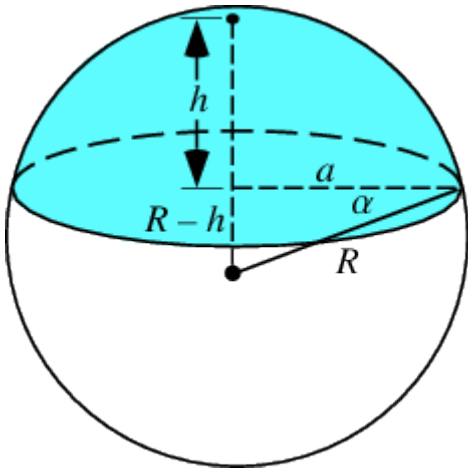
(b) the y-axis

(c) the vertical line x = 3

(d) the horizontal line y = -1

(e) The horizontal line y =2

5. Consider the portion of the ball of radius R centered at the origin for $y \geq R - h$ where $0 < h < R$. Find the volume of this spherical cap.



6. Consider the region $\mathbb{R}$ bounded by the curves $y = x^2$ and $y = 2 - x$. Find the volume of the solid obtained by rotating $\mathbb{R}$ about axis $x = -3$.

7. Let $\mathbb{C}$ be the region bounded by the lines $y = x$, $y = 2x$ and $y = 2$. Find the volume of the solid obtained by rotating C about the x-axis.

8. Consider the region $\mathbb{A}$ bounded by the curve $y = x^2 - x^3$ and the x-axis. Find the volume obtained by rotating $\mathbb{A}$ about:

(a) the y-axis
(b) the vertical line $x = 1$
(c) the vertical line $x = 3$
(d) the vertical line $x = -3$

9. The region in the xy-plane defined by the inequalities $0 \leq x \leq 2$ and $x^2/4 \leq y \leq 1$ is rotated about the given axis below. Find the volume of the solid of revolution so generated.

(a) the x-axis

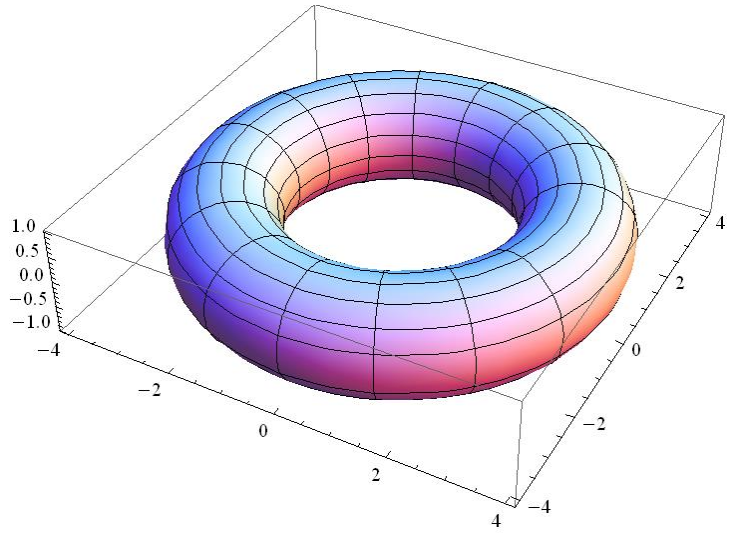
(b) the y-axis

(c) the vertical line $x = 2$

(d) the horizontal line $y = 1$

10. Find the volume of the *torus* obtained by revolving the disk $x^2 + y^2 \leq a^2$ about the line $x = b$,

where $b > a$.



Bonaventura Francesco Cavalieri (1598 – 1647)