WORKSHEET XX

PARTIAL FRACTION DECOMPOSITION

1. Find the *partial fraction decomposition* of each of the following rational functions. (To check your answers in Mathematica, use the *Apart* command.)

(a)
$$\frac{x+3}{(x+1)(x+2)}$$

(b)
$$\frac{x^4+1}{x(x-2)}$$

(c)
$$\frac{1}{(x+5)(x-2)(x-3)}$$

(d)
$$\frac{x^2+1}{(x+4)^2(x-2)}$$

(e)
$$\frac{x+3}{(x+5)(x-2)(x-3)}$$

2. Find the *form* of the partial fraction decomposition of each of the following. You need not solve for the constants.

(a)
$$\frac{x+8}{(x+5)^3(x-2)^2(x-3)}$$

(b)
$$\frac{3x}{(x^2+5)(x^2-7x+10)}$$

(c)
$$\frac{x^5 + x + 1}{(x^2 + x + 5)^2 (x + 2010)}$$

(d)
$$\frac{x^{11} + 4x^4 + 1}{(x^2 + 1)^3(x - 1)(x + 2)^4}$$

3. Compute the *indefinite integral* of each of the following functions.

(Warning: first convert to a rational function!)

(a)
$$\frac{1}{e^{2x}-2e^x-35}$$

$$(b) \frac{\sin x}{\cos^2 x + \cos x - 20}$$

$$(c) \quad \frac{1}{x(\ln x)(1+(\ln x)^2)}$$

We live but a fraction of our lives.

- Henry David Thoreau

This is a tricky domain because, unlike simple arithmetic, to solve a calculus problem - and in particular to perform integration - you have to be smart about which integration technique should be used: integration by partial fractions, integration by parts, and so on.

- Marvin Minsky