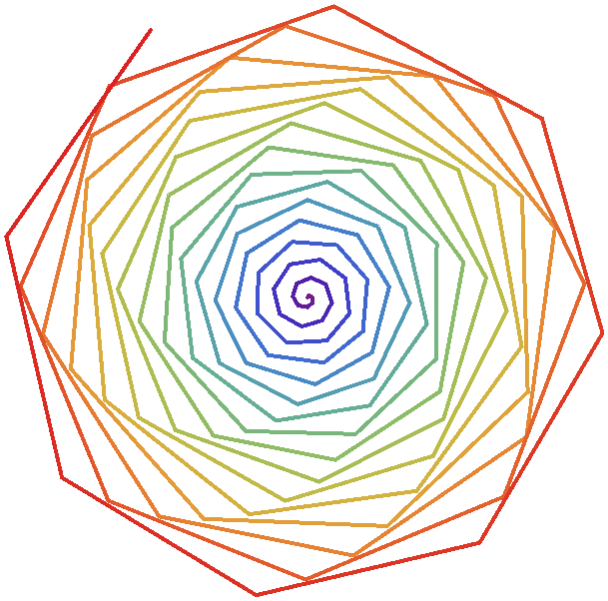
WORKSHEET IV

PARAMETRIC EQUATIONS – A BRIEF INTRODUCTION



1. Sketch the curve $x(t) = 3t$, $y(t) = t^2 + 1$. Express y as a function of x.
2. Describe the parameterized curve $x(t) = 3 \cos t$, $y(t) = 4 \cos t$, $0 \le t \le 2\pi$.

What is the relationship between the given curve and each of the following?

(a) $x(t) = -3 \cos t$, $y(t) = 4 \cos t$, $0 \le t \le 2\pi$.

(b) $x(t) = 3 \cos 2t$, $y(t) = 4 \cos 2t$, $0 \le t \le 2\pi$.

(c) $x(t) = 1 - 3 \cos 2t$, $y(t) = 1 - 4 \cos 2t$, $0 \le t \le 2\pi$.

3. Show that the following is a parameterization of the cycloid:

   $x(\theta) = a(\theta - \sin \theta)$, $y(\theta) = a(1 - \cos \theta)$, $-\infty < \theta < \infty$.

4. Show that $x = a \cos t + h$, $y = b \sin t + k$, $0 \le t \le 2\pi$, is a parametric equation of an ellipse with center at (h, k) and axes of length 2a and 2b.
5. Find a parameterization of the straight line $y = 3x + 4$.
6. Find a parameterization of the straight line segment joining the points P = (3, 5) to Q = (7, 11).
7. Find a parameterization of the curve $y = x^2$ from P = (-1, 1) to Q = (4, 16).

8. Generalize problem 7 for any curve of the form $y = f(x)$ from $x = a$ to $x = b$.