WORKSHEET VIII

LITTLE OH AND BIG OH

Suppose that $f(x) \to \infty$ and $g(x) \to \infty$ as $x \to \infty$. We say that "*f is of smaller order than g*" if $\frac{f(x)}{g(x)} \to 0$ as $x \to \infty$. In this case we write $f = o(g)$.

Assume that *f* and *g* are each positive for large *x*. We say that "*f is at most the order of g*" if there is a positive integer *M* for which $\frac{f(x)}{g(x)} \leq M$ for large x. In this case we write $f = O(g)$.

Determine which of the following statements are true; justify each answer.

(a) $3x^2 + 11 = o(x^5 + x + 99)$

(b) $x + 5 \sin x = O(x)$

(c) $2^x = o(x^{100})$

(d) $3^x = O(e^x)$

(e) $x = o(\ln x)$

(f) $3 + \ln x + \ln(\ln x) + \sqrt{x} = o\left(x^{\frac{2}{3}}\right)$

(g) $\ln x = o\left(\sqrt{x}\right)$

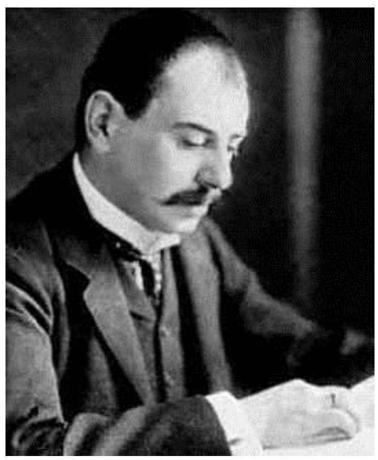
(h) $(x^2+1)^4 = O((2x+1)^3 x^5)$

(i) $\dfrac{x^2+13x+2015}{5x+1789} = O\left(\sqrt{x^2+9}\right)$

(j) $\ln x = o(\ln(\ln x))$

(k) $\ln(x^{55}+x^{33}+x^{11}+1) = O(\ln x)$

(l) $(\ln x)^{100} = o(x^{1/25})$



Edmund Landau (1877 – 1938) is known for his work in analytic number theory and the distribution of primes. He first introduced the *little oh* notation in 1909.