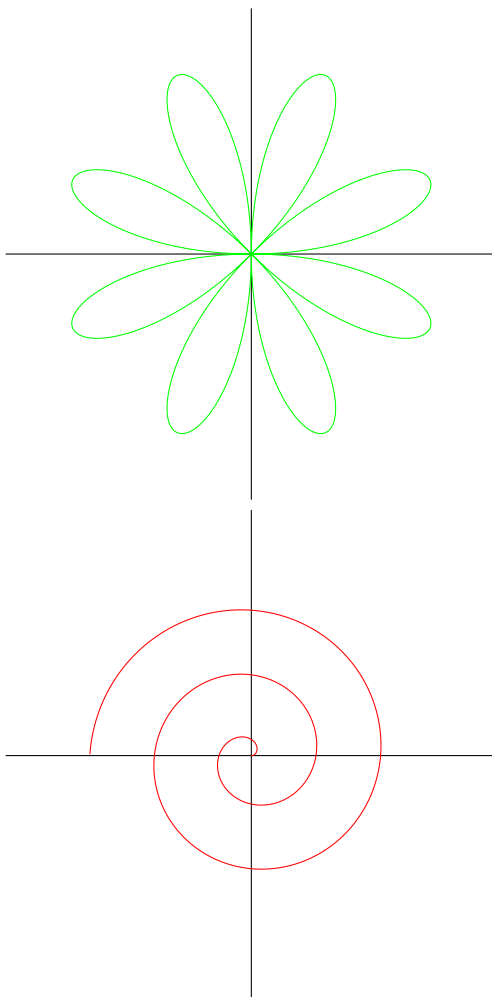


MATH 116 - TEAM HOMEWORK 3
WINTER 2016

- (1) It is time for the final of the 23rd World Martial Arts Tournament. After two defeats, O-guk is here for yet another time. His opponent, Junior, has malevolent plans for when he is declared the world's most powerful being. This fight is for the fate of the Earth! When it starts, the two opponents power up. Their bodies each produce an aura in the shape given by the following polar equations: $r = 70 \sin(4\theta)$, $0 \leq \theta \leq 2\pi$ for Junior and $r = 4\theta$, $0 \leq \theta \leq 5\pi$ for O-guk. Their power level is given by the area of the region enclosed by those curves. Will Junior bring terror to the Earth or will O-guk save it and become the world's strongest fighter?

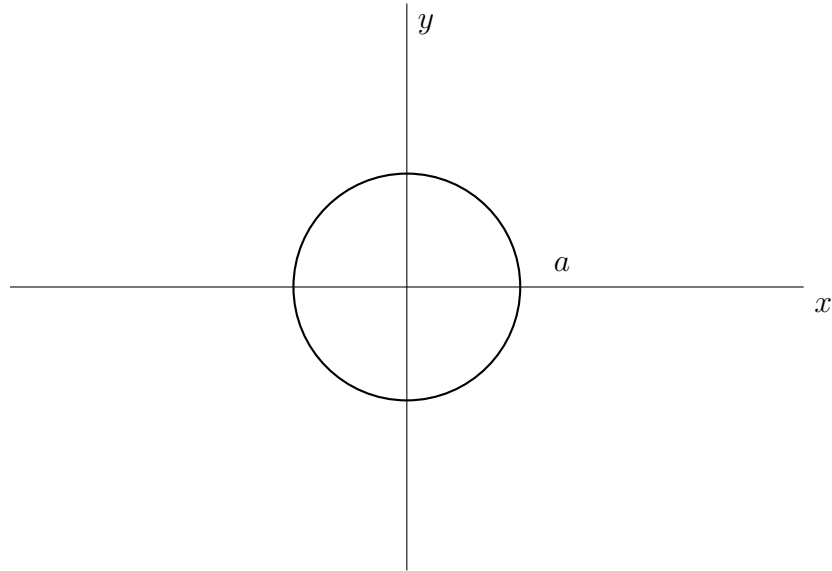


(2) Consider the following parametric equations

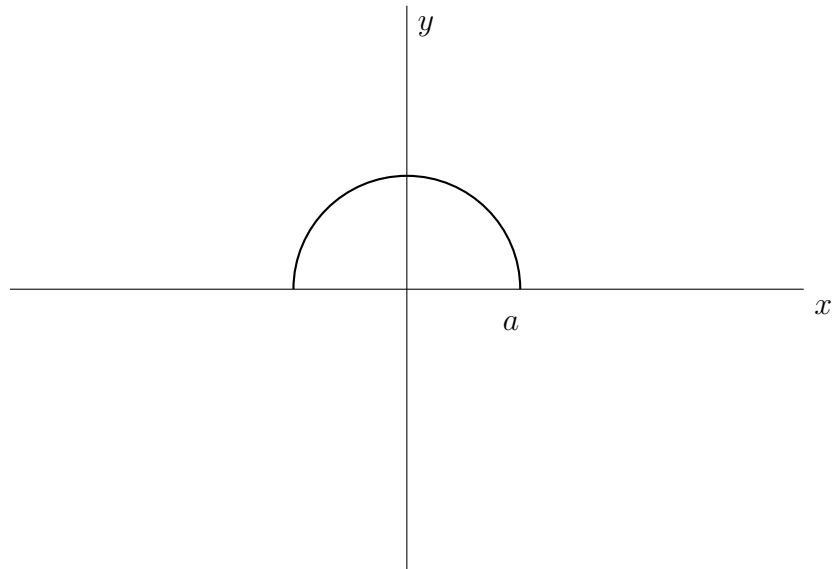
$$x(t) = \cos(t) \qquad y(t) = \cos(2t)$$

- (a) Use your calculator to draw on the plane the curve these equations define. Describe what kind of shape it is. Justify your answer using algebra.
- (b) Find the arc length of this curve. (Do it in more than one way if possible.)
- (c) Find the equation of the tangent line to the curve at time $t = \frac{\pi}{4}$.

- (3) (a) Write equations in Cartesian, parametric and polar coordinates for the circle of radius $a > 0$ centered at the origin. Be sure to include an appropriate range of values for t and θ with your equations.



- (b) Write equations in Cartesian, parametric and polar coordinates for the semicircle of radius $a > 0$ centered at the origin which lies above the x -axis. Be sure to include an appropriate range of values for t and θ with your equations.



(4) Consider the differential equation

$$y' + 2xy = 1$$

(a) Show that there is no quadratic polynomial that satisfies the equation.

(b) Show that each member of the following family of functions satisfies the equation:

$$y(x) = \frac{\int_0^x e^{t^2} dt + C}{e^{x^2}}$$