MATH 116 - TEAM HOMEWORK 4
WINTER 2016

(1) A spaceship just landed on the earth. The door opens and someone comes out. Could it be? Is this another monkey? His name is Prince V, he loves vegetables as much as O-guk and his purpose is to rule the Earth with a sword of injustice! O-guk is the Earth's only hope. Since his win at the last World Martial Arts Tournament, he has trained for the last two years. The function $V(x)$, which gives Prince's V power level, is the solution to the differential equation

$$y' = y \cos x$$

with $V(0) = 100$. The function $G(x)$, which gives O-guk's power level, is the solution to the differential equation

$$xy' = 4y$$

with $G(1) = 17$. The fighter whose power has the largest value for $0 \leq x \leq 2$ will be victorious. Determine whether Prince V will bring terror to the Earth or O-guk will save the day, once again.

(2) A can of orange juice is taken out of a 30^o F refrigerator and placed in a 70^o F room.

(a) Write a differential equation for T, the temperature of the can at time t minutes later.

(b) Find the equilibrium solution of the differential equation. Determine from the equation whether it is a stable or an unstable equilibrium.

(c) After one hour we measure the temperature of the can at 50^o F. What will it be after one more hour?

(3) A certain population of insects is growing at a rate proportional to the square of the population. The existence of a new species of frogs in the area decreases the population of the insects at a steady rate of 100 insects per day.

(a) Write a differential equation for the population of the insects in thousands as a function of time in days using constant of proportionality k.

(b) Assume that $k = 7$. One solution curve of the differential equation passes through the point $(0, 0)$. Use Euler's method with $\Delta x = 1$ to approximate three more points on that curve.

(4) Consider the differential equation

$$y' = xe^y + x^3$$

(a) Write a differential equation that involves y'', x, y but not y'.

(b) Show that all solution curves of the differential equation are concave up.