MATH 162 GW III (SOLIDS) 27 JANUARY 2016

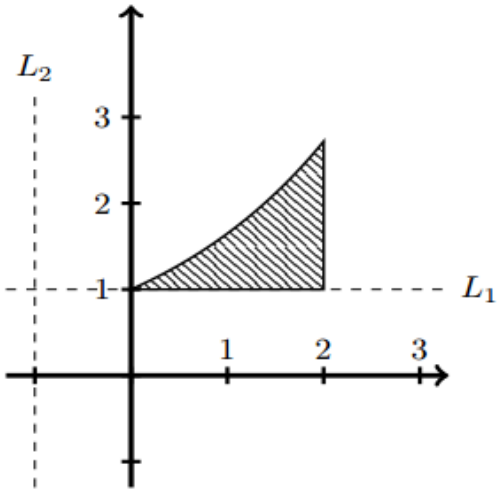
U. Michigan exam problems

Problem 1

[4 points] Let S be the solid generated by rotating the area enclosed by the curves $y = x^2$ and $y = \sqrt{x}$ around the line $x = 1$. Set up, but do not evaluate a definite integral that can be used to compute the volume of S.

Problem 2

[12 points]
The region bounded by the graph of $y = e^{0.5x}$, the line $y = 1$, and the line $x = 2$ is shown below. For each of the lines L_1 and L_2 write a definite integral that represents the volume of the solid object obtained by rotating the region around that line. You do not need to show your work or calculate the value of the integral.

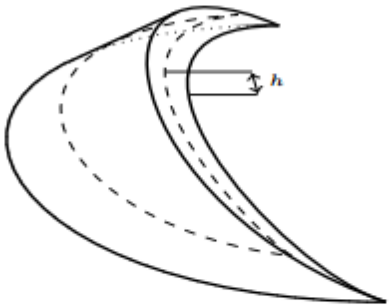


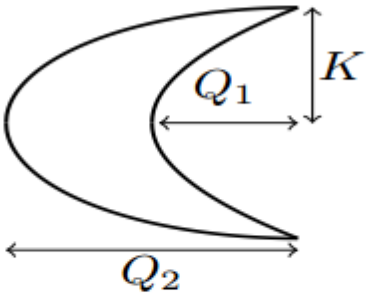
a. [6 points] L_1:

b. [6 points] L_2:

Problem 3

[12 points] Sand dunes come in many shapes. *Barchan* dunes, which have the shape shown on the left, are studied extensively by geomorphologists. Horizontal cross-sections of these dunes are crescent-shaped (the dashed line encloses one such cross-section), and can be approximated as the shape on the right. The area of this shape is given by the formula $A_h = K(\frac{\pi}{2}Q_2 - \frac{4}{3}Q_1)$.





You are studying a barchan dune of 10 meters height, for which the values of Q_1, Q_2, and K vary with respect to the height h (in meters) of the cross-section according to the functions $Q_1(h) = 10-h$, $Q_2(h) = 20-2h$, $K(h) = 100-h^2$. The density of sand in the dune is $\delta = 1600$ kilograms per cubic meter.

a. [5 points] Write an expression for the volume of one slice of sand dune h meters above the ground and Δh meters thick.

b. [5 points] Write a definite integral that represents the total mass of sand in the dune. You do not need to evaluate this integral.