*Problem 1*

[12 points]  Country music legend Willie Nelson is concerned about our dependence of fossil fuels. In 2005, he started a company which sells a bio-diesel fuel called BioWillie. He recently added a new cylindrical underground storage tank at his factory, and he needs to know how much work is required to pump all the fuel in a full tank to the surface. The tank is pictured below. It is 5 meters long and has a radius of 2 meters. Its center line is 17 meters underground. BioWillie fuel has a density of 900 kg per cubic meter. Make sure to include appropriate units and justification to support your answers.
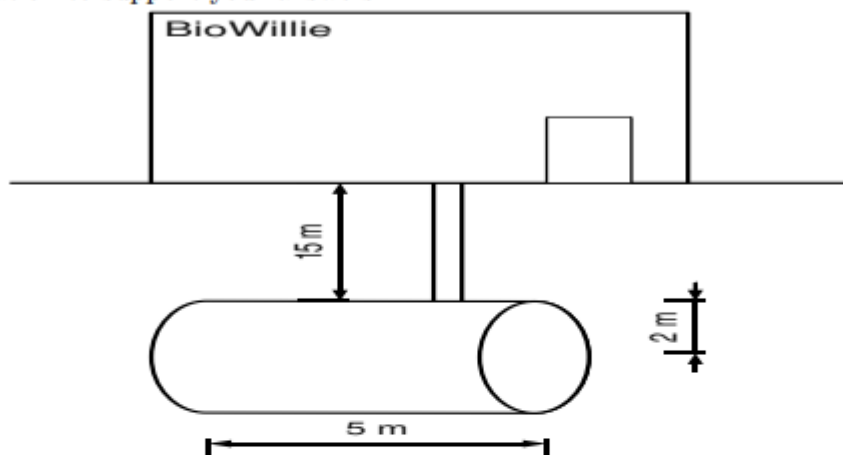


a. [7 points]  Write an expression that approximates the work done in lifting a horizontal slice of fuel that is $h_i$ meters below the ground's surface, given that the thickness of the slice is $\Delta h$ meters.

b. [5 points]  Help Willie Nelson by computing the work required to pump all the fuel in a full tank to the ground's surface. You can use your calculator to compute your final answer.

## Problem 2

Recall the quiz 2 problem.  Now compute the work done in digging the hole.

[10 points] As part of an exploration assignment, a team of mining engineers dug a hole in the ground. The hole takes the shape of a solid region of known cross-section. The hole is narrow at its bottom and becomes wider as the height rises toward ground level.  The base region, which stands vertically, is pictured below. Cross-sections taken perpendicular to the $y$-axis are squares with one side lying on the $xy$-plane.

3D view of the hole
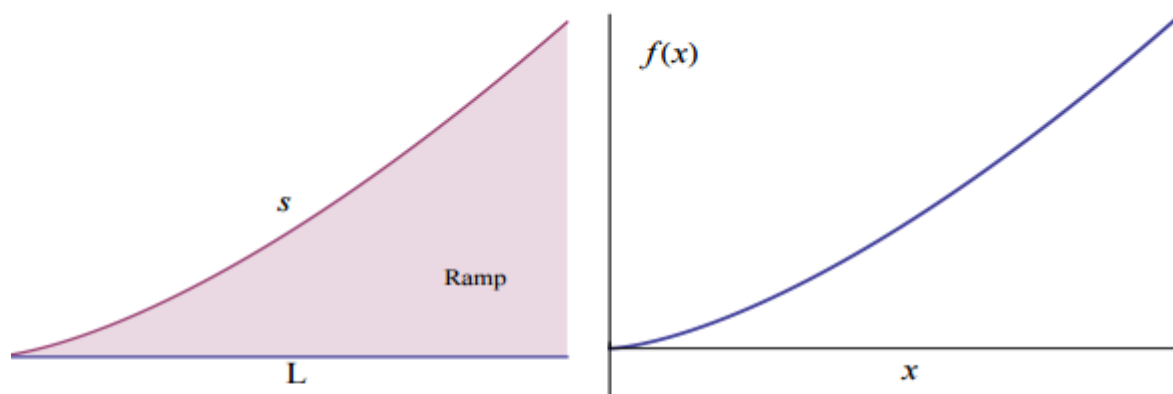
Hole

$y = \sqrt{x-1}$
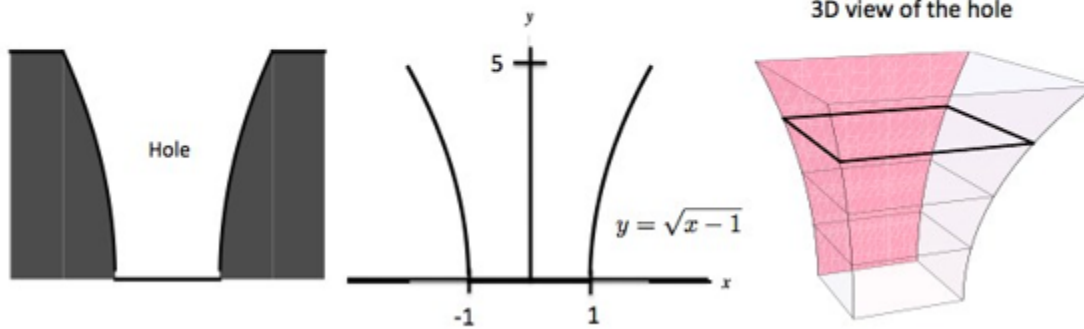
The variables $x$ and $y$ are given in meters.

The density of the soil is 1600 kg

## Problem 3

[8 points]  A company wants to design a bicycle ramp using the shape of the graph of the function $f(x) = \frac{4}{3}x^{\frac{3}{2}}$, where $x$ is the length in meters of the base of the ramp.



Find the length $s$ of a ramp with base of length $L$. Show all your work.

[11 points]  The lateral faces of a tank are determined by the curve $y = 1 - x^4$ and the $x$-axis (where $x$ and $y$ are measured in meters).  The length of the tank is 10 meters.  Be sure to include units in your answers.
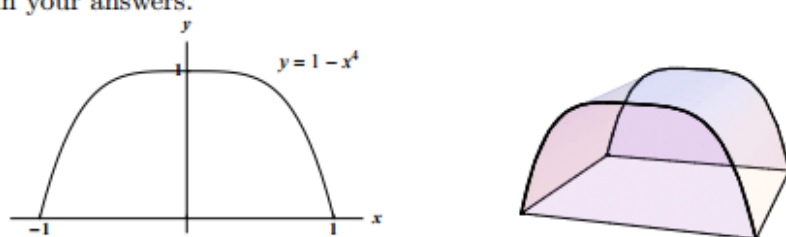


**a.** [5 points]  The tank is filled with water to a height of one half a meter. If the density of water is $1,000 \text{ kg/m}^3$, write an expression that approximates the mass of one slice of water $y$ meters above the ground and $\Delta y$ meters thick.

**b.** [2 points]  Write a definite integral that represents the total mass of water in the tank.

**c.** [4 points]  Write a definite integral that represents the amount of work required to pump the water to the top of the tank.