MATH 162 GW VI 29 FEBRUARY 2016

Problems chosen from University of Michigan final exams.

Problem 1

[14 points] For each of the following sequences

1. Compute $\lim_{n\to\infty} a_n$.
2. Decide if $\sum_{n=0}^{\infty} a_n$ converges or diverges. Circle your answer.

Support your answer by stating the test(s) or facts you used to prove convergence or divergence , and show complete work and justification.

a. [4 points]

$$a_n = \left(\frac{-1}{\pi}\right)^n \qquad \lim_{n\to\infty} a_n = \_\_\_\_\_ \qquad \sum_{n=0}^{\infty} a_n : \quad \text{Converges} \quad \text{Diverges}$$

b. [4 points]

$$a_n = \frac{n^2+2}{1+4n^2} \qquad \lim_{n\to\infty} a_n = \_\_\_\_\_ \qquad \sum_{n=0}^{\infty} a_n : \quad \text{Converges} \quad \text{Diverges}$$

c. [6 points]

$$a_n = \frac{n}{\sqrt{n^4+5}} \qquad \lim_{n\to\infty} a_n = \_\_\_\_\_ \qquad \sum_{n=0}^{\infty} a_n : \quad \text{Converges} \quad \text{Diverges}$$

Problem 2

[7 points] For $n \geq 1$, consider the following sequences

- $a_n = (-1)^n + \frac{1}{n}$.
- $b_n = 1 + \frac{(-1)^n}{n}$.
- $c_n = (\frac{6}{5})^n$.
- $s_n = \sum_{k=1}^{n} \frac{1}{k^2}$.

Circle your answers. No justification is needed.

1. Which sequences are bounded? a_n b_n c_n s_n None.

2. Which sequences are increasing? a_n b_n c_n s_n None.

3. Which sequences are convergent? a_n b_n c_n s_n None.

Problem 3

[12 points] Determine whether the following series converge or diverge (circle your answer). Be sure to mention which tests you used to justify your answer. If you use the comparison test or limit comparison test, write an appropriate comparison function.

a. [3 points] $\sum_{n=1}^{\infty}(-1)^n \frac{\sqrt{n}}{1+2\sqrt{n}}$

b. [4 points] $\sum_{n=1}^{\infty} ne^{-n^2}$

c. [5 points] $\sum_{n=1}^{\infty} \frac{\cos(n^2)}{n^2}$