Lissajous figure parameterized by

x(t) = 4 sin (9t), y(t) = 7 sin (8t + π/2),

where $0 \leq t \leq 2\pi$

1.   Find a parameterization of the circle centered at C = (7, 11) that has radius equal to 4. Choose the *clockwise* direction.

2. How many *complete cycles* will Charlotte make if she lives on the following parameterized curve: x(t) = 5 cos 20π   y(t) = 5 sin 20π where $0 \leq t \leq 1$?

3.   Find a parameterization of the line segment beginning at P = (-3, 4) and terminating at Q = (9, 9).

4.   Sketch and identify the curve defined by the parametric equations:
$$x(t) = 1 + 13 \cos t, \quad y = 3 + 13 \sin t, \quad \text{where } 0 \leq t \leq \pi/2 \text{ ?}$$

5.   Parameterize *one cycle* of the curve y = sin 14x.

6.   Sketch and identify the curve defined by the parametric equations:
$$x(t) = t^2 - t, \quad y(t) = 3t - 1$$

7. Find the arc length of each of the following three curves:

$y = \ln(1 - x^2), \quad 0 \leq x \leq \tfrac{1}{2}$

$y = e^x, \quad 0 \leq x \leq 1$

$y = \ln\left(\dfrac{e^x + 1}{e^x - 1}\right), \quad a \leq x \leq b, \quad a > 0$

8. Find a curve through the point (e, 4) in the xy-plane whose arc length from x = 1 to x = 7 is given by:

$$\int_1^7 \sqrt{1 + \frac{(\ln x)^2}{x^2}}\ dx$$

9. Charlotte, the spider, lives on the xy-plane. At time t (minutes), she is located at $x(t) = \cos t$ and $y(t) = \sin^2 t$ (units in meters) where distance is measured in yards. *How far* does Charlotte travel from $t = 0$ to $t = \pi/2$ minutes? *(You need not evaluate the integral.)*

10. Find the area of the surface obtained by rotating the curve $y = 2\sqrt{1-x},\ -1 \le x \le 0$, about the *x-axis*. (You need not evaluate the integral.) Sketch.

11. set up an integral to find the

area of the surface generated by rotating the curve $y = \sqrt{x}$, $0 \le x \le 4$, about the line $y = 4$.

12. The curve $y = \sqrt{4 - x^2},\ -1 \le x \le 1$, is an arc of the circle $x^2 + y^2 = 4$.

Find the area of the surface obtained by rotating this arc about the x-axis. (The surface is a portion of a sphere of radius 2.

13. Find the surface area obtained when the parameterized curve $x = \sin t,\ y = t^2 + t,\ 0 \le t \le \pi$ is rotated about (a) the x-axis, (b) the y-axis, (c) the line x = 9, (d) the line y = -1.

14.

Find the area of the surface obtained by rotating the circle
$x^2 + y^2 = r^2$ about the line $y = r$.

15.   An inverted conical tank of height 20 feet and base radius of 10 feet is filled with olive oil weighing 51 lb/ft^3.   How much work does it take to pump all of the oil to the rim of the tank?

16.   A swimming pool has the shape shown below:
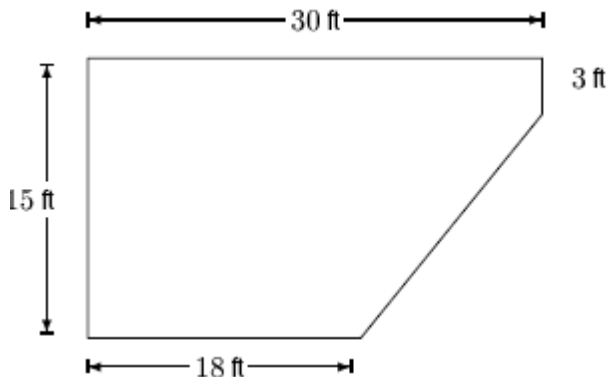
If the pool is 13 ft. deep, how much work is done in pumping all the water out? (*Note:* Water weighs 62.4 lbs per cubic foot.)

17.   The Great Cone of Alphaville was built by high school students during their summer vacation. The cone is 100 feet high and its base has a diameter of 80 ft. It has been built from bricks (made of straw) which weigh 3 lbs/ft^3. Express as a Riemann integral the amount of work
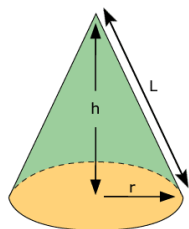

done in building the Great Cone.

18.   A chlorine solution is poured over the surface of a rectangular swimming pool that is 20 meters long, 13 meters wide, and 2 meters deep everywhere. Before the circulating pumps are turned on, it is discovered that the density of the chlorine solution at a height h meters above the bottom of the pool is given by $\rho(h) = 100(2 - h)$ gm/m^3. (That is, the chlorine solution's density is greater near the bottom of the pool.)

  (a) Express the total mass of chlorine in the pool as a Riemann sum.

  (b) Transform the Riemann sum of part (a) into a definite integral that gives the total mass of the chlorine solution in the pool. Evaluate the integral.

19. The density of cars (in cars per kilometer) down a 20 km stretch of the Auto-Route near Bordeaux is given by $\rho(x) = 600 + 120 \sin(\pi x)$ where x is the distance in miles from the toll plaza and $0 \le x \le 20$.

(a) Write a Riemann sum that estimates the total number of cars along this 20 km stretch.

(b) Convert this sum to a Riemann integral and evaluate it.

20. Match the graphs of the parametric equations $x = f(t)$ and $y = g(t)$ in (a)–(d) with the parametric curves labeled I–IV. Give reasons for your choices.

(a)

I

(b)

II

(c)

III

(d)

IV
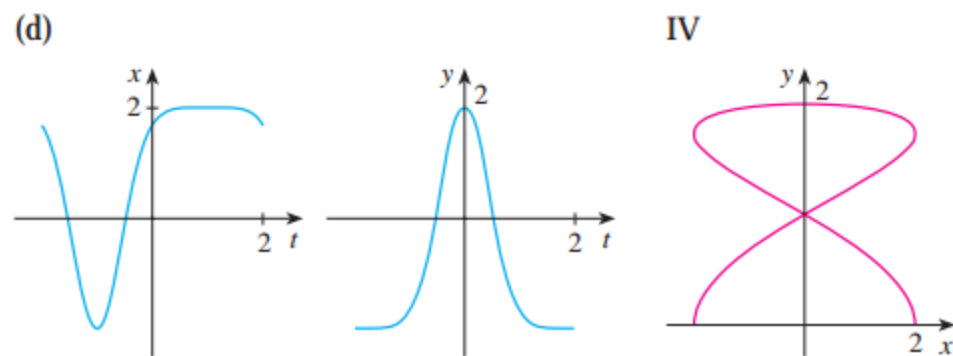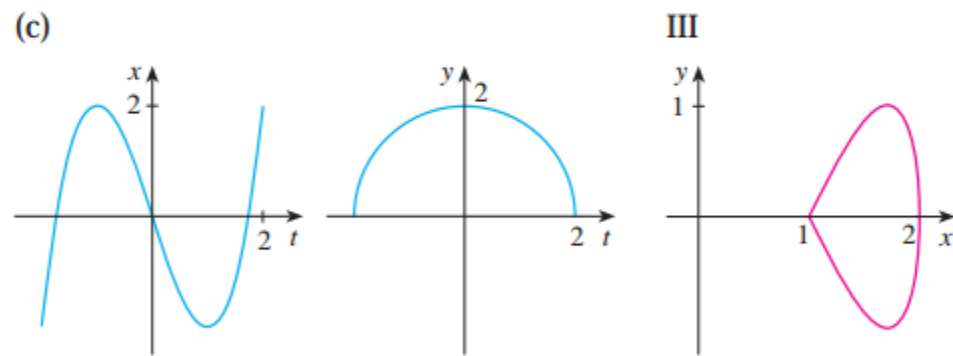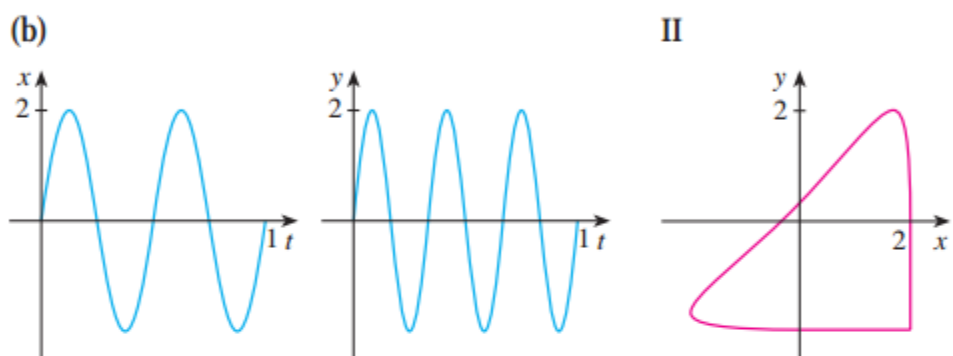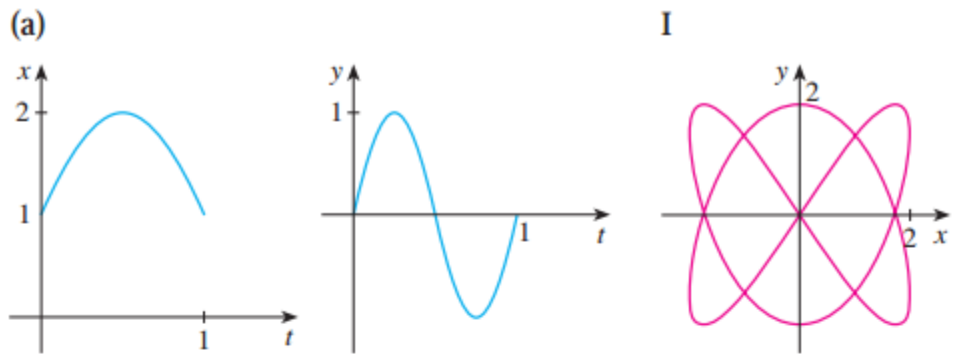
21. Match the parametric equations with the graphs labeled I-VI. Give reasons for your choices. (Do not use a graphing device.)

(a) $x = t^4 - t + 1, \quad y = t^2$
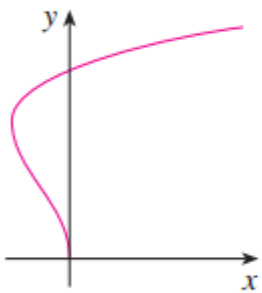(b) $x = t^2 - 2t, \quad y = \sqrt{t}$
(c) $x = \sin 2t, \quad y = \sin(t + \sin 2t)$
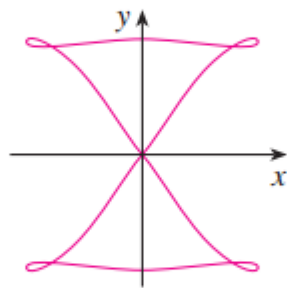(d) $x = \cos 5t, \quad y = \sin 2t$
(e) $x = t + \sin 4t, \quad y = t^2 + \cos 3t$
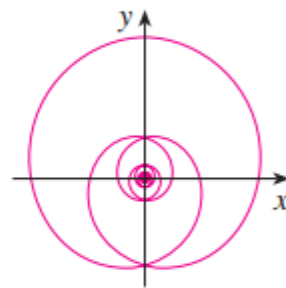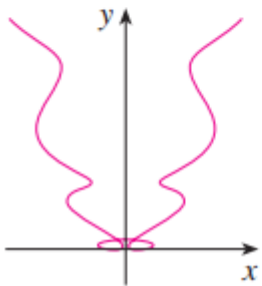(f) $x = \dfrac{\sin 2t}{4 + t^2}, \quad y = \dfrac{\cos 2t}{4 + t^2}$
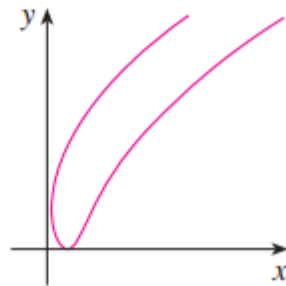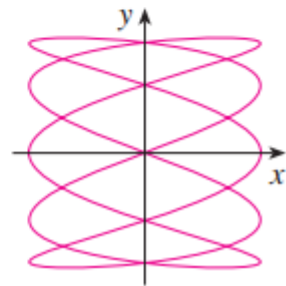
I

II

III

IV

V

VI

[12 points] Country music legend Willie Nelson is concerned about our dependence of fossil fuels. In 2005, he started a company which sells a bio-diesel fuel called BioWillie. He recently added a new cylindrical underground storage tank at his factory, and he needs to know how much work is required to pump all the fuel in a full tank to the surface. The tank is pictured below. It is 5 meters long and has a radius of 2 meters. Its center line is 17 meters underground. BioWillie fuel has a density of 900 kg per cubic meter. Make sure to include appropriate units and justification to support your answers.
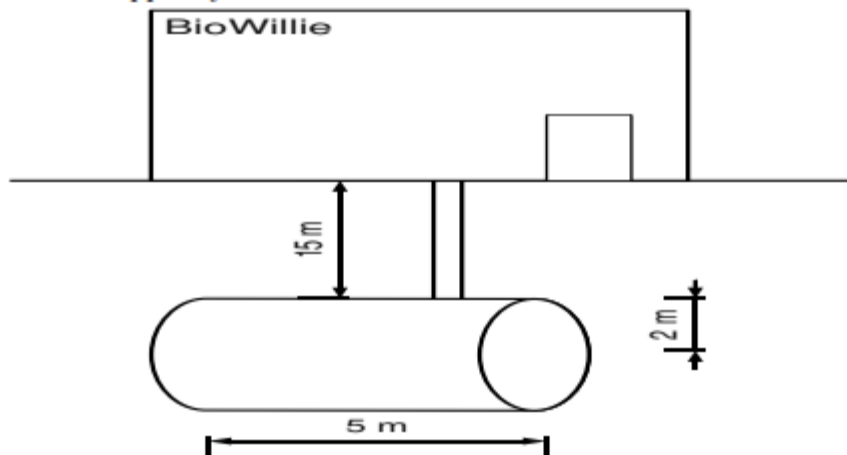

a. [7 points] Write an expression that approximates the work done in lifting a horizontal slice of fuel that is h_i meters below the ground's surface, given that the thickness of the slice is Δh meters.

b. [5 points] Help Willie Nelson by computing the work required to pump all the fuel in a full tank to the ground's surface. You can use your calculator to compute your final answer.

Recall the quiz 2 problem.   Now compute the work done in digging the hole.

[10 points] As part of an exploration assignment, a team of mining engineers dug a hole in the ground. The hole takes the shape of a solid region of known cross-section. The hole is narrow at its bottom and becomes wider as the height rises toward ground level. The base region, which stands vertically, is pictured below. Cross-sections taken perpendicular to the y-axis are squares with one side lying on the xy-plane.

3D view of the hole

$y = \sqrt{x - 1}$
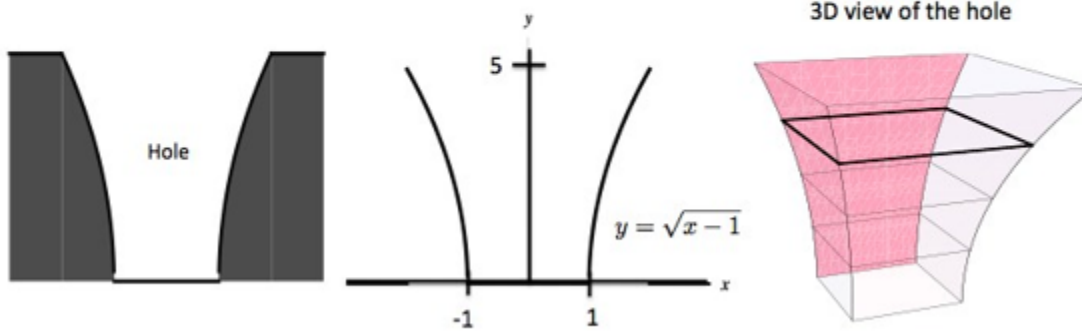
The variables x and y are given in meters.

The density of the soil is 1600 kg

The limits of my language are the limits of my world.

- Wittgenstein, **Tractatus Logico-Philosophicus**