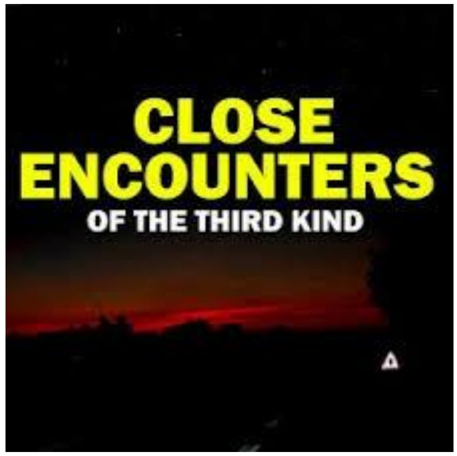
MATH 162 PRACTICE QUIZ IV



1. Compute the *exact value* of each of the following convergent improper integrals:

$$(a)\quad \int_0^{\infty} e^{-5x}\ dx$$

$$(b)\quad \int_0^{\infty} xe^{-x^2}\ dx$$

$$(c)\quad \int_e^{\infty} \frac{1}{x(\ln x)^4}\ dx$$

$$(d)\quad \int_{e^e}^{\infty} \frac{1}{x(\ln x)(\ln \ln x)^{1.1}}\ dx$$

$$(e)\quad \int_{0+}^{1} \frac{1}{\sqrt{x}}\ dx$$

$$(f)\quad \int_0^{1-} \frac{1}{\sqrt{1-x^2}}\ dx$$

$$(g)\quad \int_0^{\infty} xe^{-x}\ dx$$

2. For each of the following improper integrals, determine convergence or divergence. *Justify your answers!*

$$(a)\quad \int_e^{\infty} \frac{1}{\ln x}\ dx$$

$$(b)\quad \int_e^{\infty} \frac{1}{x^{1/3}(\ln x)^4}\ dx$$

$$(c)\quad \int_0^{\infty} \sqrt{x}\ e^{-x^2}\ dx$$

3. For each of the following improper integrals, determine *convergence or divergence. Justify each answer! (That is, if you use the comparison test, exhibit the function that you choose to use for comparison and show why the appropriate inequality holds.)*

(a) $$\int_0^{\infty} \frac{1+x+x^4}{(1+x)^5}\ dx$$

(b) $$\int_0^{\infty} \frac{1+x+e^x}{5+3e^{3x}}\ dx$$

$$(c)\quad \int_e^{\infty} \frac{1+x+x^2+2016x^{99}}{1+(\ln x)^{91}+(x^5+1)^{21}}\ dx$$

4. For each improper integral given below, determine convergence or divergence. (You may either perform the integration directly or else use the Comparison Test.) *Justify your answers!*

(a) $$\int_0^\infty e^{-3x}\,dx$$

(b) $$\int_{19}^\infty \frac{x^3}{x^4+33}\,dx$$

(c) $$\int_{71}^\infty \frac{1}{\sqrt{x+13}}\,dx$$

(d) $$\int_3^\infty \frac{1}{(x+9)^{\frac{5}{4}}}\,dx$$

(e) $$\int_5^\infty \frac{1}{x(\ln x)\ln(\ln x)}\,dx$$

(f) $$\int_5^\infty \frac{1}{x(\ln x)(\ln(\ln x))^{1.01}}\,dx$$

5. For each improper integral given below, determine convergence or divergence. (You will need to use the *Comparison Test* here.) *Justify your answers!*

(a) $$\int_0^{\infty} \frac{\sin^4 x}{(1+x)^2}\, dx$$

(b) $$\int_4^{\infty} \frac{1}{(\ln x)-1}\, dx$$

(c) $$\int_0^{\infty} \frac{(3+x)^2 + x\ln x + 5x + 1}{(2016+13x+x^2)^4}\, dx$$

(d) $$\int_1^{\infty} \frac{\ln x}{x^3}\, dx$$

6. Find the *volume* of the solid of revolution obtained by rotating the curve $y = 1/(1 + x^2)^{1/2}$ from $x = 0$ to $x = \infty$ about the x-axis or explain why no such number exists.

7. For each of the following improper integrals, determine *convergence or divergence*. Use an appropriate version of the *Comparison Test*.

(a) $$\int_{0+}^{\infty} \frac{1}{x^{\frac{2}{3}} + x^{\frac{4}{3}}}\, dx$$

(b) $$\int_{0+}^{\infty} \frac{1+x}{x^3 + \sqrt{x}}\, dx$$

(c) $$\int_0^{\frac{\pi}{2}-} \tan x\, dx$$

(d) $$\int_{0+}^{1} \frac{1-\ln x}{x^4}\, dx$$

8. For each of the following improper integrals, determine *convergence* or *divergence*. *Justify your answers!*

(a) $$\int_{0+}^{\infty} \frac{1+x}{x^3+\sqrt{x}}\, dx$$

(b) $$\int_{0}^{\frac{\pi}{2}-} \tan x \; dx$$

(c) $$\int_{0}^{2-} \frac{1}{\sqrt{4-x^2}}\, dx$$

(d) $$\int_{0+}^{1} \frac{1-\ln x}{x^4}\, dx$$

9. (*Thomas*) Let T be the lifetime in years of an Oz Cell Phone. Assume that this lifetime is modeled with the following exponential density function

$$f(T) = \begin{cases} 0 \; if \quad T < 0 \\ 0.1e^{-0.1T} \; if \;\; T \geq 0 \end{cases}$$

(a) Verify that this function is a probabilty density function.

Using this model,

(b) Find the probability that a cell phone will last for *more than 2 years.*

(c) Find the probability that a cell phone will fail in the 4^{th} year.

10. Find the value of c so that $f(x)=\begin{cases} f(x)=c\sqrt{x}\,(1-x) & if\ \ 0\le x\le 1 \\ 0 & otherwise \end{cases}$

is a probability density funtion.

11. For each improper integral (of the second kind) below, determine convergence or divergence. For those that converge, compute its value.

$$(a)\quad \int_{0+}^{1} \frac{1}{\sqrt{x}}\,dx$$

$$(b)\quad \int_{0}^{1-} \frac{1}{\sqrt{1-x^2}}\,dx$$

$$(c)\quad \int_{0}^{\frac{\pi}{2}-} \sec x\,dx$$

$$(d)\quad \int_{0+}^{e} \ln t\,dt$$

12. Find the *volume* of the solid of revolution obtained by rotating the curve $y = 1/x^2$ from x = 1 to x = ∞ about the x-axis or explain why no such number exists.

13. *(University of Michigan)*

[15 points] Each of the integrals below are improper. Determine the convergence or divergence of each. Make sure you include all the appropriate steps to justify your answers. Approximations with your calculator will not receive credit.

a. [4 points]

$$\int_1^\infty \frac{5-2\sin x}{\sqrt{x^3}}dx$$

b. [5 points]

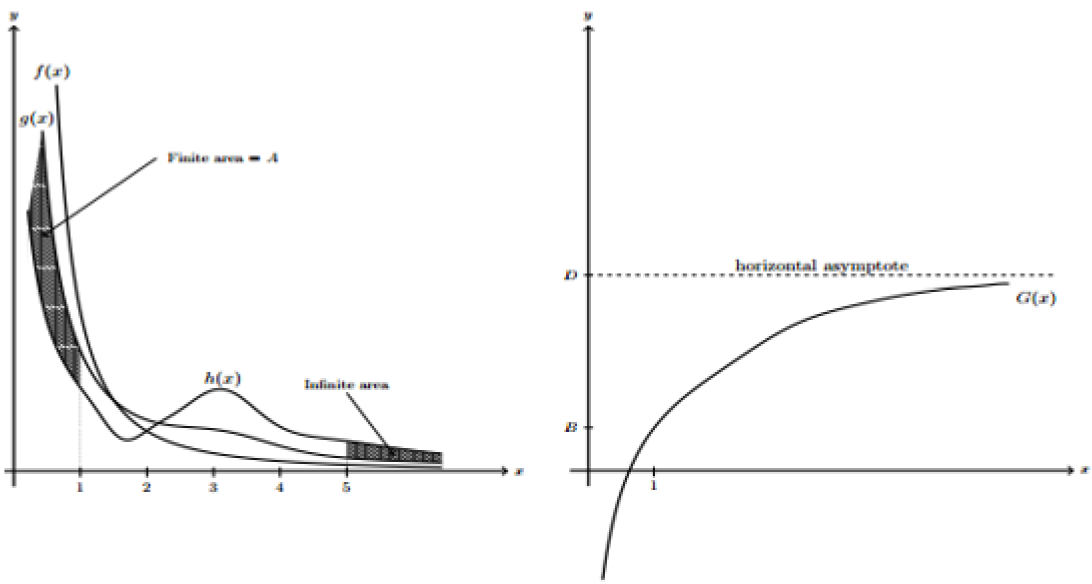
$$\int_1^2 \frac{x^2}{(x^3-1)^2}dx$$

c. [6 points]

$$\int_2^\infty \frac{1}{(x^3+7)^{\frac{1}{3}}}dx$$

14. *(University of Michigan)*

[15 points] Graphs of f, g and h are below. Each function is positive, is continuous on $(0,\infty)$, has a horizontal asymptote at $y = 0$ and has a vertical asymptote at $x = 0$. The area between $g(x)$ and $h(x)$ on the interval $(0, 1]$ is a finite number A, and the area between $g(x)$ and $h(x)$ on the interval $[5,\infty)$ is infinite. On the right is a graph of an antiderivative $G(x)$ of $g(x)$. It also has a vertical asymptote at $x = 0$.
Use the information in these graphs to determine whether the following three improper integrals **converge**, **diverge**, or whether there is **insufficient information to tell**. You may assume that f, g and h have no intersection points other than those shown in the graph. **Justify all your answers.**



a. [3 points] $\int_1^\infty h(x)dx$

b. [4 points] $\int_0^1 g(x)dx$

c. [3 points] $\int_0^1 h(x)dx$

d. [5 points] If $f(x) = 1/x^p$, what are all the possible values of p? **Justify your answer.**

As far at the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality.

- Albert Einstein, **Sidelights on Relativity**