MATH 162 PRACTICE QUIZ V

1. For each improper integral (of the second kind) below, determine convergence or divergence. For those that converge, compute its value.

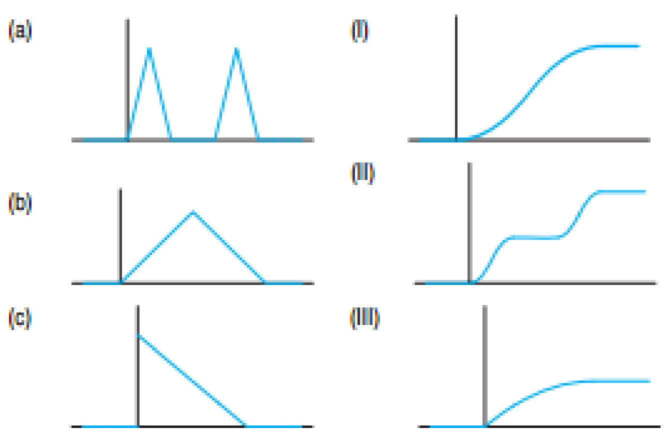
(a) $\displaystyle\int_{0+}^{1} \frac{1}{\sqrt{x}}\, dx$

(b) $\displaystyle\int_{0}^{1-} \frac{1}{\sqrt{1-x^2}}\, dx$

(c) $\displaystyle\int_{0}^{\frac{\pi}{2}-} \sec x \ dx$

2.

Match the graphs of the density functions (a), (b), and (c) with the graphs of the cumulative distribution functions I, II, and III.



3. In each of the following two exercises, graph a density function and a cumulative distribution function which could represent the distribution of income through a population with the given characteristics.

(a) A large middle class.

(b) Small middle and upper classes and many poor people.

(c) Small middle class, many poor and many rich people.

4. Find the *volume* of the solid of revolution obtained by rotating the curve $y = 1/x^2$ from $x = 1$ to $x = \infty$ about the x-axis or explain why no such number exists.

13.

. The density function and cumulative distribution function of heights of grass plants in a meadow are in Figures 8.95 and 8.96, respectively.

(a) There are two species of grass in the meadow, a short grass and a tall grass. Explain how the graph of the density function reflects this fact.

(b) Explain how the graph of the cumulative distribution function reflects the fact that there are two species of grass in the meadow.

(c) About what percentage of the grasses in the meadow belong to the short grass species?

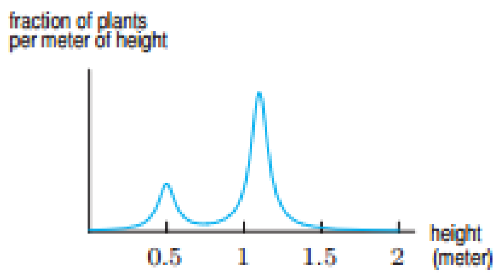


Figure 8.95

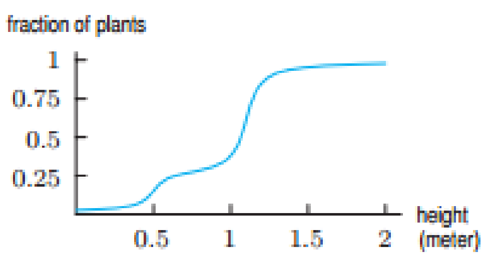
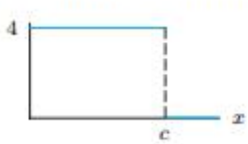
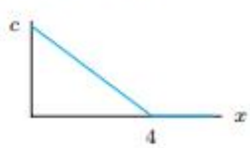
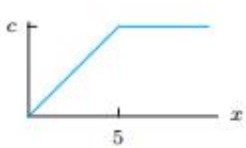


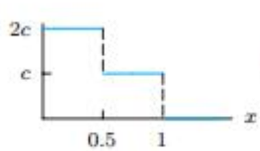
Figure 8.96

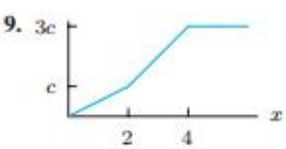
Decide if the function graphed in Exercises 5–10 is a probability density function (pdf) or a cumulative distribution function (cdf). Give reasons. Find the value of c. Sketch and label the other function. (That is, sketch and label the cdf if the problem shows a pdf, and the pdf if the problem shows a cdf.)

5. 

6. 

7. 

8. 

9. 

10. 

11. Let $p(x)$ be the density function for annual family income, where x is in thousands of dollars. What is the meaning of the statement $p(70) = 0.05$?

12. Find a density function $p(x)$ such that $p(x) = 0$ when $x \geq 5$ and when $x < 0$, and is decreasing when $0 \leq x \leq 5$.

14. For each of the following improper integrals (of the second or third kind), determine convergence or divergence. As needed, use an appropriate version of the Comparison Test.

(a) $$\int_{0+}^{\infty} \frac{1}{x^{\frac{2}{3}} + x^{\frac{4}{3}}} \, dx$$

(b) $$\int_{0+}^{\infty} \frac{1+x}{x^3 + \sqrt{x}} \, dx$$

(c) $$\int_{0}^{\frac{\pi}{2}-} \tan x \, dx$$

(d) $$\int_{0+}^{1} \frac{1-\ln x}{x^4} \, dx$$

(e) $$\int_{0}^{\frac{\pi}{2}-} \tan x \, dx$$

(f) $$\int_{0}^{2-} \frac{1}{\sqrt{4-x^2}} \, dx$$

(g) $$\int_{0+}^{1} \frac{1-\ln x}{x^4} \, dx$$

15. For each of the following sequences, determine *convergence* or *divergence*. In the case of convergence, find the *limit* of the sequence. *Briefly explain your reasoning!*

(a) $c_n = \sqrt{1 - \frac{3}{n}}$

(b) $d_n = \frac{\cos^4(n^3 + n^2 + 5)}{n^2 + 23}$

(c) $e_n = 1 + \arctan n$

(d) $u_n = \frac{e^n}{n^5}$

(e) $z_n = \frac{(n^5 + 1)^3 (1 - 4n)^2}{(n + 9)^{17}}$

16. Consider the following *recursively defined* sequence:

$a_1 = 4$

$a_2 = 2$

$a_n = a_{n-1}a_{n-2} - a_{n-1} - a_{n-2} + 1$ for $n \geq 3$.

Find the numerical values of a_3, a_4, a_5 and a_6. (Show your work.)

17. Why does an increasing sequence that is bounded above converge? Is there a similar result for decreasing sequences?

18. Does every bounded sequence converge? Can a convergent sequence have two limits?

19. For $n \geq 1$, let

$$a_n = \int_0^1 (x^2 + 2)^n \, dx.$$

Determine convergence or divergence of the sequence $\{a_n\}$. (*Hint:* Do *not* try to evaluate the integral! Calculator solutions are not accepted.)

Hint: Is the sequence *monotone*?

20. Let $a_n = 1/1 + ½ + 1/3 + ¼ + \ldots + 1/n$ for $n \geq 1$(integers only)

Demonstrate that the sequence $\{a_n\}$ diverges.

21. Assuming that the limit exists, find it.

$$a_1 = 1 + \sqrt{2},\ a_2 = 1 + \sqrt{1 + \sqrt{2}},\ a_3 = 1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{2}}}}, \ldots$$

22. By computing the first few terms, guess what the limit of the following recursively defined sequence.

$$a_1 = 1,\ a_n = \frac{1}{2}\left(a_{n-1} + \frac{5}{a_{n-1}}\right)\ \textit{for}\ n \geq 2$$

There is more danger of numerical sequences continued indefinitely than of trees growing up to heaven. Each will some time reach its greatest height.

- Friedrich Ludwig Gottlob Frege, **Grundgesetz der Arithmetik** (1893)