MATH 162 (REVISED) PRACTICE QUIZ VI

1. For each of the following sequences, determine *convergence* or *divergence*. In the case of convergence, find the *limit* of the sequence. *Briefly explain your reasoning!*

(a) $c_n = \sqrt{1 - \dfrac{3}{n}}$

(b) $d_n = \dfrac{\cos^4(n^3 + n^2 + 5)}{n^2 + 23}$

(c) $e_n = 1 + \arctan n$

(d) $u_n = \dfrac{e^n}{n^5}$

(e) $z_n = \dfrac{(n^5 + 1)^3 (1 - 4n)^2}{(n + 9)^{17}}$

2. Consider the following *recursively defined* sequence:

$a_1 = 4$

$a_2 = 2$

$a_n = a_{n-1}a_{n-2} - a_{n-1} - a_{n-2} + 1$ for $n \geq 3$.

Find the numerical values of a_3, a_4, a_5 and a_6. (Show your work.)

3. To which of the following series does the "*n^{th} term test for divergence*" apply? Explain!

$$(a)\quad \sum_{n=1}^{\infty} \frac{n}{n+5}$$

$$(b)\quad \sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^{n}$$

$$(c)\quad \sum_{n=1}^{\infty} \frac{1}{n}$$

$$(d)\quad \sum_{n=1}^{\infty} \frac{1}{n^3}$$

$$(e)\quad \sum_{n=1}^{\infty} \arctan(n)$$

$$(f)\quad \sum_{n=1}^{\infty} n^{1/n}$$

4. For $n \geq 1$, let

$$a_n = \int_0^1 (x^2+2)^n \, dx.$$

Determine convergence or divergence of the sequence $\{a_n\}$. (*Hint:* Do *not* try to evaluate the integral! Calculator solutions are not accepted.)

Hint: Is the sequence *monotone*?

5. Let $a_n = 1/1 + ½ + 1/3 + ¼ + \ldots + 1/n$ for $n \geq 1$(integers only)

Demonstrate that the sequence $\{a_n\}$ diverges.

6. Assuming that the limit exists, find it.

$$a_1 = 1+\sqrt{2},\ a_2 = 1+\sqrt{1+\sqrt{2}},\ a_3 = 1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{2}}}},\ldots$$

7. By computing the first few terms, guess what the limit of the following recursively defined sequence.

$$a_1 = 1,\ a_n = \frac{1}{2}\left(a_{n-1} + \frac{5}{a_{n-1}}\right)\ \ for\ \ n \geq 2$$

8. Carefully state the **Comparison Test** for positive series.

9. For each of the following infinite series, determine *convergence* or *divergence*. *In the case of convergence, find the sum of the series:*

$(a)\ \sum_{n=1}^{\infty} \ln\frac{n+1}{n}$

(b) $\sum_{n=0}^{\infty} \frac{5}{9^n}$

$(c)\ \sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^n$

(d) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ (Hint: Calculate the first few partial sums.)

(e) $\sum_{n=1}^{\infty} \cos\left(\frac{5}{n}\right)$

(f) 0.123123123…

10. For each sequence below, determine *convergence* or *divergence*. Justify your answers. Calculator results will not earn full credit.

(a) $a_n = \dfrac{100^n + 1789^n}{n! + 7^n}$

(b) $b_n = \left(1 + \dfrac{4}{n}\right)^n$

(c) $c_n = \dfrac{\ln(n + 2016\pi)}{\ln(n)}$

(d) $d_n = \dfrac{\cos\left(\dfrac{\pi}{n}\right)}{n}$

(e) $e_n = \displaystyle\int_0^n e^{-\pi t}\, dt$

(f) $f_n = \sqrt{\dfrac{n+1}{n} + \dfrac{\sin\left(n^2\right)}{n^2} + e^{-\frac{3}{n}} + \dfrac{e^n}{\sinh n}}$

(g) $g_n = \dfrac{(\ln \ln n)^{2525}}{n}$

11. (a) Use the comparison test to show that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges. (*Hint:* Compare to the telescoping series $\sum_{n=2}^{\infty} \frac{1}{n(n-1)}$.)

(b) What can you say about $\sum_{n=1}^{\infty} \frac{1}{n^p}$ where p ≥ 2? (Hint: *Compare* to $\sum_{n=1}^{\infty} \frac{1}{n^2}$)

(c) What can you say about $\sum_{n=1}^{\infty} \frac{1}{n^p}$ where p ≤ 1?

(*Hint:* Compare to $\sum_{n=1}^{\infty} \frac{1}{n}$)

12. Find the *sum* of each of the following convergent series. Show your work.

(a) $\sum_{n=0}^{\infty} \left(e^{-n} - e^{-n-1}\right)$

(b) $\sum_{k=0}^{\infty} \frac{(-4)^{k+1}}{5^{k-1}}$

(c) 5.314314314314314…

13.

[4 points] The series

$$\sum_{n=0}^{\infty} \frac{9^n}{8^n + 10^n}$$

converges.

Use an appropriate series test to show that the series converges. Be sure to indicate which test(s) you are using. Also verify all hypotheses needed for the test, and justify the convergence/divergence of any other series you use.

14. Does the following series converge or diverge? Justify.

$$\sum_{n=4}^{\infty} \frac{1}{n^3 + n^2 \cos(n)}$$

15.

[11 points] Determine the convergence or divergence of the following series. In parts (a) and (b), support your answers by stating and properly justifying any test(s), facts or computations you use to prove convergence or divergence. Circle your final answer. Show all your work.

a. [3 points] $\displaystyle\sum_{n=1}^{\infty} \frac{9n}{e^{-n} + n}$ CONVERGES DIVERGES

There is more danger of numerical sequences continued indefinitely than of trees growing up to heaven. Each will some time reach its greatest height.

- Friedrich Ludwig Gottlob Frege, **Grundgesetz der Arithmetik** (1893)