MATH 162 PRACTICE QUIZ VII

1. For each numerical series below, determine *convergence* or *divergence*. In the case of convergence, determine if the series converges *absolutely* or *conditionally*. Justify each answer.

$$(a)\quad \sum_{m=1}^{\infty}\left(\frac{-e}{m}\right)^{m}$$

$$(b)\quad \sum_{n=1}^{\infty}\frac{(-1)^{3n}2^{n}}{\left(1+\frac{1}{n}\right)^{n^2}}$$

$$(c)\quad \sum_{n=3}^{\infty}(-1)^{n}\frac{5}{\ln n}$$

$$(d)\quad \sum_{n=1}^{\infty}\frac{(-1)^{n}(n!)^{2}3^{n}}{(2n+1)!}$$

$$(e)\quad \sum_{k=1}^{\infty}(-1)^{k}\frac{k(k+1)(k^{2}+5)}{(k-13\ln k)^{4}}$$

$$(f)\quad \sum_{n=1}^{\infty}\frac{5^{n}+7}{11^{n}}$$

$$(g)\quad \sum_{n=1}^{\infty}\frac{(-1)^{n+1}(3n)!}{(n!)^{3}}$$

$(h)\ \sum_{n=1}^{\infty} (-1)^n \frac{4^n (n!)^2}{(2n)!}$

2. For each of the following numerical series, determine if the series *diverges*, *converges conditionally* or *converges absolutely*. Justify your answers!

(a) $\sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{1}{5+n^7 \ln n}\right)$

(b) $\sum_{n=2}^{\infty} \frac{\sin(3n+5)}{n(\ln n)^2}$

(c) $\sum_{n=3}^{\infty} (-1)^n \frac{5}{\ln n}$

(d) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{1+n}{5+n^2}\right)^n$

(e) $\sum_{k=1}^{\infty} (-1)^k \frac{k(k+1)(k^2+5)}{(k-13\ln k)^4}$

(f) $$\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{\left(1+\frac{1.3}{n}\right)^{n^2}}$$

3. Does absolute convergence imply convergence? Does convergence imply absolute convergence? Why?

4. How many terms are required in order to estimate each of the following sums accurately to 4 decimal places?

(a) $$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2}$$

(b) $$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^8}$$

(c) $$\sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln n}$$

5. For each of the following *power series*, determine the *interval of convergence*. Do not study end-point behavior.

(a) $$\sum_{n=1}^{\infty} \frac{3^n}{n(n+5)} x^n$$

(b) $$\sum_{n=1}^{\infty} \frac{1}{n(n+3)(n+11)} (x-4)^n$$

(c) $\sum_{n=1}^{\infty} \frac{n^n}{n!}(x-1)^n$

(d) $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}(x-4)^n$

(e) $\sum_{n=0}^{\infty} \frac{1}{2^n \sqrt{n+1}}(x+3)^n$

(f) $\sum_{n=1}^{\infty} \frac{7^n}{n^5 e^n} x^n$

6. *(University of Michigan final exam)*

[6 points] The power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{5^n n}$$

has a radius of convergence of 5. For each of the endpoints of the interval of convergence, fill in the first two blanks with the endpoint and the series at that endpoint (in sigma notation or by writing out the first 4 terms), and then indicate whether the series converges at that endpoint in the final blank. You **do not** need to show your work.

At the endpoint $x =$______________, the series is ____

and that series ____________________________.

At the endpoint $x =$______________, the series is ____

and that series ____________________________.

7. *(University of Michigan final exam)*

[6 points] Find the **radius** of convergence of the power series. You do not need to find the interval of convergence.

Consider the function $g(x)$ defined by the power series

$$g(x) = \sum_{n=0}^{\infty} \frac{2^n (n!)^2 x^n}{(2n)!}.$$

Pure mathematics is, in its way, the poetry of logical ideas.

- Albert Einstein