MATH 162 PRACTICE QUIZ VIII

1. Find a power series expansion of $\int e^{-x^2}\,dx$

2. For each of the following functions, find the 4th order Taylor polynomial centered at $x = c$:

   (a) $y = \sinh x + 3\cosh x$, $c = 0$

   (b) $y = 1 + x + e^{3x}$, $c = 0$

   (c) $y = 1/(x + 2)$, $c = 0$

   (d) $y = \ln(1 + x)$, $c = 0$

   (e) $y = x^{1/2}$, $c = 4$

   (f) $y = \sin x$, $c = \pi/4$

   (g) $y = 1 + x + 3x^2 - 4x^3$, $c = 0$

   (h) $y = 1 + x + 3x^2 - 4x^3$, $c = 1$

   (i) $y = xe^{2x}$, $c = 0$

3. Using multiplication of power series, find the *first four non-zero* terms of the Maclaurin series expansion of $f(x) = e^{2x}\cos(3x)$.

4. Using division of power series, find the *first four non-zero* terms of the Maclaurin series expansion of

$$G(x) = \frac{\cosh x}{1 + x + x^2}$$

5. Using your choice of technique, find the *first four non-zero* terms of the Maclaurin series expansion of:

   (a) $y = xe^{-4x}$

   (b) $y = (2 + x)/(1 - x)$

(c) $y = (1 - x - x^2)\, e^{2x}$

(d) $y = (\sin x) \ln(1 + x)$

(e) $y = x \cos^2 x$

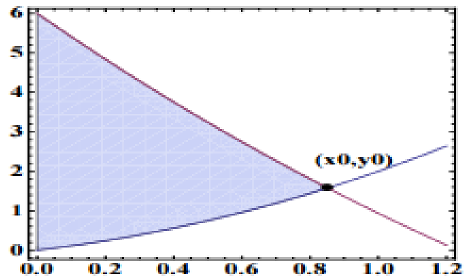
(f) $y = e^{x^3}$

(g) $y = \exp(1 + x^2)$

6. Find the Taylor series expansion of $y = e^x$ at $x = c$.

7. (University of Michigan final exam problem)

[15 points] The graph shows the area between the graphs of $f(x) = 6\cos(\sqrt{2x})$ and $g(x) = x^2 + x$. Let (x_0, y_0) be the intersection point between the graphs of $f(x)$ and $g(x)$.



a. [6 points] Compute $P(x)$, the function containing the first three nonzero terms of the Taylor series about $x = 0$ of $f(x) = 6\cos(\sqrt{2x})$.

b. [3 points] Use $P(x)$ to approximate the value of x_0.

c. [3 points] Use $P(x)$ and the value of x_0 you computed in the previous question to write an integral that approximates the value of the shaded area. Find the value of this integral.

d. [1 point] Graph $f(x)$ and $g(x)$ in your calculator. Use the graphs to find an approximate value for x_0.

e. [2 points] Write a definite integral in terms of $f(x)$ and $g(x)$ that represents the value of the shaded area. Find its value using your calculator.

8. Without using L’Hôpital’s rule, find

$$\lim_{x \to 0} \frac{\ln(1+x) - x}{1 - \cos x}$$

9. By differentiating an appropriate power series, compute the following sum:

$$\sum_{n=1}^{\infty} \frac{n}{5^n}$$

10. Find the Taylor series of

$$F(x) = \cos\sqrt{x+1}$$

centered at x = -1.

11. Let F(x) = x^4 arctan(3x). Find $F^{(2345)}(0)$.

Hint: Beginning with a geometric series, find the Maclaurin series expansion of arctan(t).

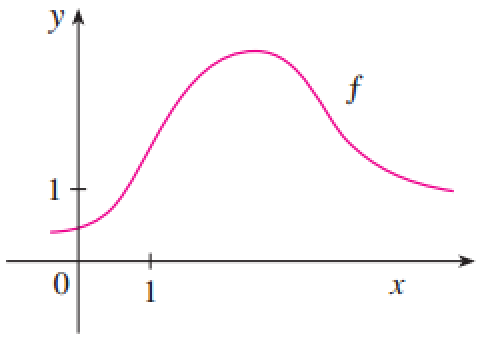
12. Without using L'Hôpital's rule, find

$$\lim_{x\to 0} \frac{e^{2x} - 4\cos\left(x^2\right) - 2x + 5 - 2e^{x^2}}{\sin(x^3) + x^5 e^x}$$

13. Find the first four non-zero terms in the Maclaurin expansion of f(x) = tan x by dividing the series for sin x by the series for cos x.

Problems from Stewart's calculus:

The graph of f is shown.



(a) Explain why the series

$$1.6 - 0.8(x-1) + 0.4(x-1)^2 - 0.1(x-1)^3 + \cdots$$

is *not* the Taylor series of f centered at 1.

(b) Explain why the series

$$2.8 + 0.5(x-2) + 1.5(x-2)^2 - 0.1(x-2)^3 + \cdots$$

is *not* the Taylor series of f centered at 2.

If the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$ is 10, what is the radius of convergence of the series $\sum_{n=1}^{\infty} nc_n x^{n-1}$? Why?

Suppose you know that the series $\sum_{n=0}^{\infty} b_n x^n$ converges for $|x| < 2$. What can you say about the following series? Why?

$$\sum_{n=0}^{\infty} \frac{b_n}{n+1} x^{n+1}$$

(a) Use differentiation to find a power series representation for

$$f(x) = \frac{1}{(1 + x)^2}$$

What is the radius of convergence?

(b) Use part (a) to find a power series for

$$f(x) = \frac{1}{(1 + x)^3}$$

(c) Use part (b) to find a power series for

$$f(x) = \frac{x^2}{(1 + x)^3}$$

(a) Find a power series representation for $f(x) = \ln(1 + x)$. What is the radius of convergence?

(b) Use part (a) to find a power series for $f(x) = x\ln(1 + x)$.

(c) Use part (a) to find a power series for $f(x) = \ln(x^2 + 1)$.

Find the *domain* of the Bessel function

$$J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$$

Show that the Bessel function

$$J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$$

is a solution to the following differential equation.

$$x^2 J_0''(x) + xJ_0'(x) + x^2 J_0(x) = 0$$

ne cannot escape the feeling that these mathematical formulas have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers.

- Heinrich Hertz