MATH 162 PRACTICE TEST 1

1. Use integration by parts to evaluate the following integral:

$$\int (\ln x)^2 \; dx$$

2. The base of a solid is the region enclosed by y = 1/x, y = 0, x = 1, and x = 4. Every cross section of the solid taken perpendicular to the x-axis is an isosceles right triangle with its hypotenuse across the base. Express the volume of the solid as a Riemann integral. (You need not evaluate your integral.)

3. A cylindrical barrel, standing upright on its circular end, contains muddy water. The top of the barrel, which has a diameter of 3 feet, is open. The height of the barrel is 4 feet and it is ¾ filled with muddy water. The weight of the muddy water at a distance of *h* feet from the bottom of the barrel is given by w(h) = 51 + k(4-h) pounds/feet^3, where *k* is a positive constant. Find the total work done in pumping the muddy water to the top rim of the barrel. (*Note:* Evaluate your integral. Your answer will include the constant *k*.)

4. A snail crawls along the curve y = $x^{3/2}$ at a speed of 3 feet per hour. How long does it take the snail to travel from the point (1, 1) to the point (4, 8)? Give a numerical answer.

5. Using implicit differentiation, find a formula for the derivative of each of the following inverse functions:

(A) arcsinh x

(B) arccosh x

(C) arctanh x

6. A cable that weighs 2 lb/ft is used to lift 800 lb of coal up a mineshaft 500 ft deep. Find the work done. Express your answer numerically.

7. Consider the region, R, bounded by the curves $y = \ln x$, $y = 0$, and $x = 2$. Suppose that R is rotated about the line $x = -1$. Express the volume of this solid of revolution as a Riemann integral using each of the following methods. Be certain to make a sketch for each.

(A) cylindrical shells

(B) washers

8. The region in the first quadrant bounded by the curves

$$y = (1 + x)^{1/2},\ y = 0,\ x = 0, \text{ and } x = 1$$

is rotated about the x-axis. Find the surface area of this solid of revolution. Express your answer numerically.

9. Suppose that the volume of water required to fill a hollow object to a depth of *h* inches (where $0 \le h \le \pi/2$) is given by the function:

$$V(h) = 1.5h + \sin h \quad \text{cubic inches.}$$

What is the *cross-sectional area* of the object 1 inch above its base?

10. Find the area of the surface obtained by rotating the triangle with vertices $(0, 0), (2, 0), (1, 3^{1/2})$ about the axis $y = -3$.

11. Using integration by parts, evaluate

$$\int x \sec^2 x \, dx.$$

12. Suppose that an inverted conical tank is filled with kerosene (weighing 51.2 lb/ft^3) and that the tank measures 30 ft high and has radius at the top of 10 ft. Compute how

much work would be done in pumping all of the kerosene over the top of the tank. (You may express your answer as a Riemann integral.)

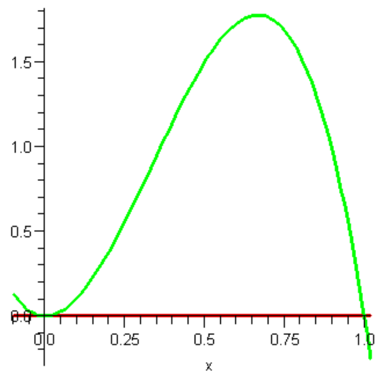
13. Express the arc length of the ellipse $\frac{x^2}{81}+\frac{y^2}{25}=1$ as a Riemann integral. (*Hint:* First parameterize the ellipse using $x = 9\cos t$ and $y = 5\sin t$ and then select an appropriate interval of t values.)

14. The base of a solid is the region between the curve $y = (\sin x)^{1/2}$ and the interval $[0, \pi]$ on the x-axis. The cross sections perpendicular to the x-axis are equilateral triangles with bases running from the x-axis to the curve $y = (\sin x)^{1/2}$. Write a Riemann integral that expresses the volume of the solid. (You need not evaluate the integral.)

15. Consider the region in the xy-plane bounded by the curves $y = 4 - x^2$ and $y = 2 - x$. If this region is rotated about the x-axis, express the volume of this solid of revolution as a Riemann integral. (You need not evaluate the integral.)

16. Consider the region in the xy-plane bounded by the curves $y = x^4$ and $y = 2$. If this region is rotated about the line $y = 13$, express the volume of this solid of revolution as a Riemann integral. (You need not evaluate the integral.)

17. Consider the region in the xy-plane bounded by the curves $y = 12(x^2 - x^3)$ and the x-axis as illustrated below. If this region is rotated about the line $x = 2$, express the volume of this solid of revolution as a Riemann integral. You need not evaluate the integral. (Hint: Use the method of cylindrical shells.)



18. Let S be the surface of revolution obtained by rotating the curve

$y = e^{-2x^2}$, $0 \le x \le 3$, about the x-axis. Find a Riemann integral that expresses the surface area of this region. (You need not evaluate the integral.)

19. A 35 foot chain with mass density 4 lb/ft is initially coiled on the ground. How much work is performed in lifting the chain vertically so that it is fully extended (with one end touching the ground)? You may express your answer as a Riemann integral.

20. Using the definitions of hyperbolic functions, verify that

$$\cosh 2x = (\cosh x)^2 + (\sinh x)^2.$$

21. Using *integration by parts*, find an anti-derivative of each if the following functions:

(a) xe^x

(b) $\sin(\ln x)$

(c) $x \sinh(3x)$

(d) arc sin x

22. Find a recursive formula for $\int x^n e^x dx$ and use it to evaluate $\int x^5 e^x dx.$

(Note that without a recursive formula, this integral would require five integration by parts in a row.)

23. Find a recursive formula for $\int \sin^n x \ dx$ and use it to evaluate $\int \sin^6 x \ dx.$

(Note that without a recursive formula, this integral would fall under "very bad case" category.)

24. The RideJoyfully company wants to design a bicycle ramp using the shape of the function f(x) = (4/3) $x^{3/2}$, where L is the length in meters of the base of the ramp. Find the length of the ramp.

There is something I don't know
that I am supposed to know.
I don't know what it is I don't know,
and yet am supposed to know,
and I feel I look stupid
if I seem both not to know it
and not know what it is I don't know.

Therefore I pretend I know it.

- R. D. Laing, **Knots**