MATH 162 SOLUTIONS: QUIZ IV

1. Find the exact value of the improper integral $\int_0^\infty \frac{x}{(x^2+13)^{3/2}}\,dx$.

Solution:

$$\int_0^\infty \frac{x}{(x^2+13)^{3/2}}\,dx = \int_0^\infty x(x^2+13)^{-3/2}\,dx = \lim_{p\to\infty}\int_0^p x(x^2+13)^{-3/2}\,dx = -\lim_{p\to\infty}(x^2+13)^{-1/2}\Big|_{x=0}^{p} =$$

$$-\lim_{p\to\infty}\left((p^2+13)^{-1/2}-(0^2+13)^{-1/2}\right) = -\lim_{p\to\infty}\left(\frac{1}{\sqrt{p^2+13}}-\frac{1}{\sqrt{13}}\right) = \frac{1}{\sqrt{13}}$$

2. Find the exact value of the improper integral $\int_0^\infty e^{-bx}\,dx$

Solution: $$\int_0^\infty e^{-bx}\,dx = \lim_{p\to\infty}\int_0^p e^{-bx}\,dx = \lim_{p\to\infty}\frac{e^{-bx}}{-b}\Big|_{x=0}^{p} = -\frac{1}{b}\lim_{p\to\infty}e^{-bx}\Big|_{x=0}^{p} =$$

$$-\frac{1}{b}\lim_{p\to\infty}e^{-bx}\Big|_{x=0}^{p} = -\frac{1}{b}\lim_{p\to\infty}\left(e^{-bp}-e^0\right) = -\frac{1}{b}\lim_{p\to\infty}\left(\frac{1}{e^{bp}}-1\right) = \frac{1}{b}$$

3. Determine whether the improper integral $\int_{16}^\infty \frac{16+x+x^2}{(1616+x)^4}\,dx$ converges or diverges. Explain your reasoning.

Solution: We begin by comparing the order of magnitude of the numerator with that of the denominator.

$$\frac{16+x+x^2}{(1616+x)^4} \approx \frac{x^2}{(1616+x)^4} \approx \frac{x^2}{x^4} = \frac{1}{x^2}$$

Since we know that $\int_{16}^\infty \frac{1}{x^2}\,dx$ converges (due to the p-test), we conjecture that our original improper integral does as well. Toward this end we invoke the Comparison Theorem.

$$0<\frac{16+x+x^2}{(1616+x)^4}<\frac{16x^2+x^2+x^2}{(0+x)^4}\approx\frac{18x^2}{x^4}=\frac{18}{x^2}$$

Now since any non-zero multiple of a convergent integral converges, it follows from the Comparison Test that our original improper integral converges.

4. *Determine whether the improper integral* $\int_{16}^{\infty}\sqrt{\frac{16+x+x^2}{(1616+x)^2}}\,dx$ *converges or diverges. Explain your reasoning.*

Solution: We begin by comparing the order of magnitude of the numerator with that of the denominator.

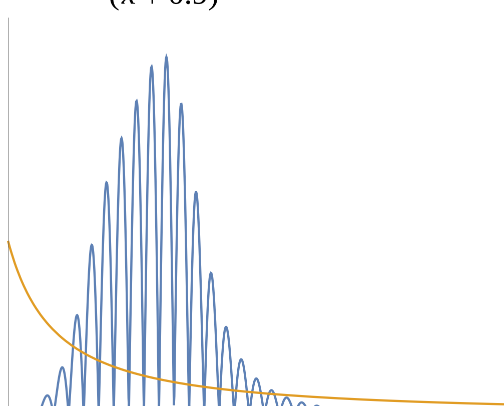
$$\sqrt{\frac{16+x+x^2}{(1616+x)^4}}\approx\sqrt{\frac{x^2}{(1616+x)^4}}\approx\sqrt{\frac{x^2}{x^4}}=\frac{1}{x}$$

Since we know that $\int_{16}^{\infty}\frac{1}{x}dx$ *diverges (due to the p-test), we conjecture that our original improper integral diverges as well. Toward this end we invoke the Comparison Theorem.*

$$\sqrt{\frac{16+x+x^2}{(1616+x)^4}}>\sqrt{\frac{x^2}{(1616x+x)^4}}=\sqrt{\frac{x^2}{(1617x)^4}}=\frac{x}{(1617)^2x^2}=\frac{1}{1617x}=\left(\frac{1}{1617}\right)\frac{1}{x}$$

Now since any non-zero multiple of a divergent integral diverges, it follows from the Comparison Test that our original improper integral diverges.

5. *Below are graphs of the function* $f(x)=\frac{1}{(x+0.9)^{0.9}}$ *and a mystery function g(x) satisfying g(0) = 0.*



(a) On the graph, label which is f(x) and which is g(x). Explain your answer in the space below.

Solution: y = f(x) is a decreasing function, f must be the orange graph.

Furthermore, since g(0) = 0, g must be the oscillating (blue) graph.

(b) Based on the graph, determine if it is possible to tell whether the improper integral $\int_{16}^{\infty} g(x)\,dx$ converges or diverges. Explain your answer.

Solution: Since $f(x) = \dfrac{1}{(x+0.9)^{0.9}} > \dfrac{1}{(1.9x)^{0.9}} > \dfrac{1}{2}\left(\dfrac{1}{x^{0.9}}\right)$

it follows from the p-test and the Comparison Test that

$\int_{16}^{\infty} f(x)\,dx$ *diverges. Since g (eventually) is dominated by f, we can draw no conclusion about the convergence or divergence of* $\int_{16}^{\infty} g(x)\,dx$

Nothing was ever achieved without enthusiasm.

- Emerson