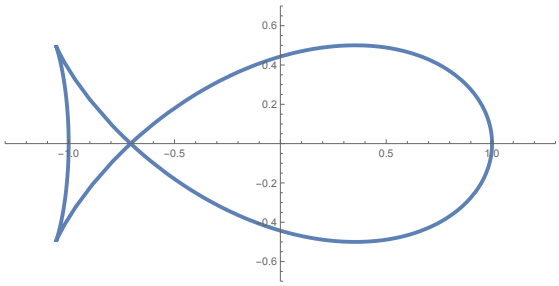
MATH 162 – Calculus II – Spring 2016
Team Homework # 1 – Due Friday, February 26

1. Recall the "Fish Curve" from Exam 1.

$$x = \cos t - \frac{\sin^2 t}{\sqrt{2}}, \qquad y = \sin t \, \cos t, \qquad 0 \le t \le 2\pi.$$



   (a) Find the coordinates of all x-intercepts of the curve. Give both the exact values and decimal approximations.

   (b) Find the coordinates of all y-intercepts of the curve. Give both the exact values and decimal approximations.

   (c) Find the slopes of the tangent lines to the curve at the "corners" of the fish's tail which are in quadrants II and III. Give both the exact values and decimal approximations.

2. Set up integrals for the lengths of the curves whose equations are:

   (a) $y = e^x$ from $x = 0$ to $x = 1$.

   (b) $x = t + \ln t, \quad y = t - \ln t$ from $t = 0$ to $t = e$.

   Show that the length of the parametric curve is equal to $\sqrt{2}$ times the length of the Cartesian curve. [Hint: You do not have to evaluate either integral to solve this problem.]

3. There is a unique real number C such that the improper integral

$$\int_1^\infty \frac{x}{2x^2 + 2C} - \frac{C}{x+1} \; dx$$

converges. Determine C and evaluate the integral at this value of C. Show that the integral diverges at all other values of C.

4. The average speed of molecules in an ideal gas is

$$\overline{v} = \frac{4}{\sqrt{\pi}} \left(\frac{M}{2RT} \right)^{3/2} \int_0^\infty v^3 e^{-Mv^2/(2RT)} \; dv$$

where M is the molecular weight of the gas, R is the gas constant, T is the gas temperature, and v is the molecular speed. Evaluate the integral to find $\overline{v}$ and simplify as much as possible.